2 Load Analysis

2.1 Introduction

This book is concerned with the design and analysis of machine and structural components. Since these are *load-carrying* members, an analysis of loads is of fundamental importance. A sophisticated stress or deflection analysis is of little value if it is based on incorrect loads. A mechanical component cannot be satisfactory unless its design is based on realistic operating loads.

Sometimes the service or operating loads can be readily determined, as are those on some engines, compressors, and electric generators that operate at known torques and speeds. Often the loads are difficult to determine, as are those on automotive chassis components (which depend on road surfaces and driving practices) or on the structure of an airplane (which depends on air turbulence and pilot decisions). Sometimes experimental methods are used to obtain a statistical definition of applied loads. In other instances engineers use records of service failures together with analyses of strength in order to infer reasonable estimates of loads encountered in service. The determination of appropriate loads is often a difficult and challenging initial step in the design of a machine or structural component.

2.2 Equilibrium Equations and Free-Body Diagrams

After certain initial applied loads have been determined or estimated, the basic equations of equilibrium enable loads at other points to be determined. For a nonaccelerating body, these equations can be simply expressed as

$$\Sigma F = 0 \quad \text{and} \quad \Sigma M = 0 \tag{2.1}$$

For an accelerating body they are

$$\Sigma F = ma \quad \text{and} \quad \Sigma M = I\alpha \tag{2.2}$$

These equations apply with respect to each of any three mutually perpendicular axes (commonly designated X, Y, and Z), although in many problems forces and moments are present with respect to only one or two of these axes.

The importance of equilibrium analysis as a means of load determination can hardly be overemphasized. The student is urged to study each of the following examples carefully as well as the vectorial approach to equilibrium balance covered in Appendix G.

SAMPLE PROBLEM 2.1 Automobile Traveling Straight Ahead at Constant Speed on Smooth, Level Road

The 3000-lb (loaded weight) car shown in Figure 2.1 is going 60 mph and at this speed the aerodynamic drag is 16 hp. The center of gravity (CG) and the center of aerodynamic pressure (CP) are located as shown. Determine the ground reaction forces on the front and rear wheels.

SOLUTION

Known: A car of specified weight travels at a given speed with known drag force.

Find: Determine the pavement forces on the tires.

Schematic and Given Data:

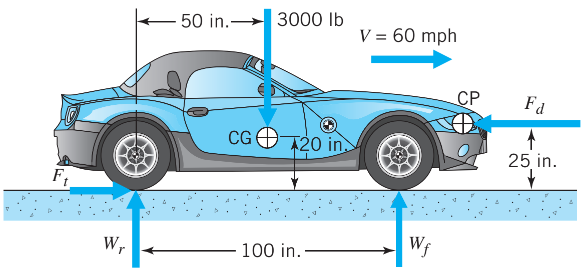


FIGURE 2.1 Free-body diagram of auto traveling at constant speed.

Assumptions:

1. The speed is constant.
2. The car has rear-wheel drive.
3. Vertical aerodynamic forces are negligible.
4. The rolling resistance of the tires is negligible.

Analysis:

1. Power is force times velocity; 1 hp = 33,000 ft·lb/min, and 60 mph = 5280 ft/min; hence,

$$\text{hp} = \frac{\text{drag force (lb)} \cdot \text{velocity (ft/min)}}{33{,}000}$$

$$16 = \frac{(F_d)(5280)}{33{,}000}$$

$$F_d = 100 \text{ lb}$$

2. Summation of forces in the direction of motion is zero (no acceleration forces exist at constant velocity); hence, thrust force F_t is 100 lb in the forward direction. This is the force applied *by* the road surface *to* the tires. (The force applied by the tires to the road is equal but opposite in direction.) This force is divided equally between the rear wheels for the rear-wheel-drive car shown; it could be applied to the front tires for a front-wheel-drive car without altering any other forces.

3. Applying the moment equilibrium equation with respect to moments about an axis passing through the rear tire road contacts, we have

$$\Sigma M = (3000 \text{ lb})(50 \text{ in.}) - (100 \text{ lb})(25 \text{ in.}) - (W_f)(100 \text{ in.}) = 0$$

from which $W_f = 1475$ lb.

4. Finally, from the summation of vertical forces equals zero, we have

$$W_r = 3000 \text{ lb} - 1475 \text{ lb}$$
$$= 1525 \text{ lb}$$

Comments: Before leaving this problem, we note two further points of interest.

1. The weight of the vehicle *when parked* is carried equally by the front and rear wheels—that is, $W_f = W_r = 1500$ lb. When traveling at 60 mph, forces F_d and F_t introduce a front-lifting couple about the lateral axis (any axis perpendicular to the paper in Figure 2.1) of 100 lb times 25 in. This is balanced by an opposing couple created by the added force of 25 lb carried by the rear wheels and the reduced force of 25 lb carried by the front wheels. (Note: This simplified analysis neglects *vertical* aerodynamic forces, which can be important at high speeds; hence, the use of "spoilers" and "wings" on race cars.)
2. The thrust force is not, in general, equal to the weight on the driving wheels times the coefficient of friction, but it *cannot exceed* this value. In this problem the wheels will maintain traction as long as the coefficient of friction is equal to or above the extremely small value of 100 lb/1525 lb, or 0.066.

Sample Problem 2.2 Automobile Undergoing Acceleration

The car in Figure 2.1, traveling 60 mph, is suddenly given full throttle. The corresponding engine power is 96 hp. Estimate the ground reaction forces on the front and rear wheels, and the acceleration of the vehicle.

Solution

Known: A car of specified weight, with known drag force and speed, is given full throttle.

Find: Determine the ground forces on the tires and the vehicle acceleration.

Schematic and Given Data:

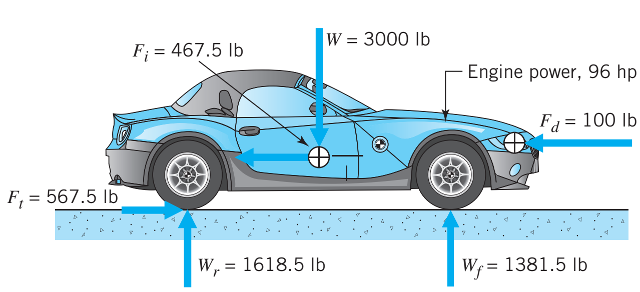


Figure 2.2 Free-body diagram of auto undergoing forward acceleration.

Assumptions:

1. The rotational inertia effect is equivalent to a car weighing 7 percent more.
2. The rear wheels develop the required traction.

Analysis:

1. The influence of the *rotating* inertia of the car wheels, engine flywheel, and other rotating members should be considered. When the car accelerates, power is consumed in *angularly* accelerating these members. Detailed calculations typically indicate that in "high" gear the effect of the rotational inertia is to increase the weight of the car by about 7 percent. This means that only 100/107 of the power available for acceleration goes to *linearly* accelerating the car mass.
2. In this problem, 16 hp gives the forward wheel thrust of 100 lb needed to maintain constant speed. With total horsepower increased to 96, 80 hp produces acceleration, of which 80(100/107) or 74.8 hp causes linear acceleration. If 16 hp produces a 100-lb thrust, then, by proportion, 74.8 hp will increase the thrust by 467.5 lb.
3. From Eq. 2.2,

$$a = \frac{F}{m} = \frac{Fg}{W} = \frac{(467.5\text{ lb})(32.2\text{ ft/s}^2)}{3000\text{ lb}} = 5.0\text{ ft/s}^2$$

4. Figure 2.2 shows the car in equilibrium. The 467.5-lb inertia force acts toward the rear and causes an additional shift of 93.5 lb from the front to the rear wheels (calculation details are left to the reader).

Comment: In this problem the wheels will maintain traction as long as the coefficient of friction is equal to or above the value of 567.5/1617, or 0.351.

SAMPLE PROBLEM 2.3 Automotive Power Train Components

Figure 2.3 shows an exploded drawing of the engine, transmission, and propeller shaft of the car in Figures 2.1 and 2.2. The engine delivers torque T to the transmission, and the transmission speed ratio $(\omega_{in}/\omega_{out})$ is R. Determine the loads, exclusive of gravity, acting on these three members.

SOLUTION

Known: An engine of known general configuration delivers power to a transmission and drive shaft of an automobile.

Find: Determine the loads on the engine, transmission, and drive shaft.

Schematic and Given Data:

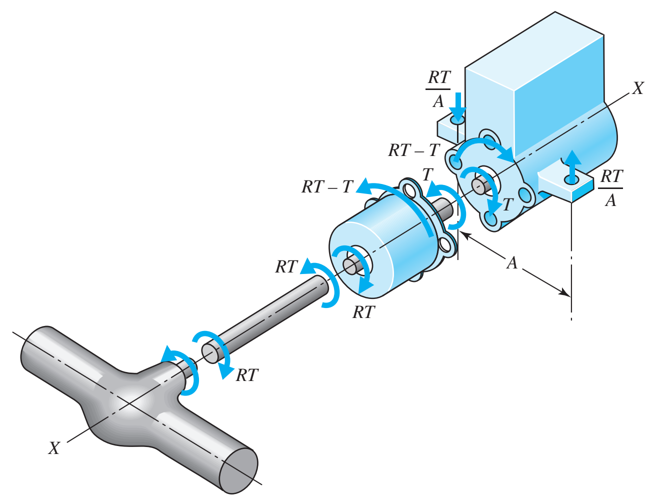


FIGURE 2.3
Equilibrium of moments about the *X* axis for engine, transmission, and propeller shaft of a front-engine, rear-wheel-drive automobile (T = engine torque, R = transmission torque ratio; engine rotates counterclockwise viewed from transmission).

Assumptions:

1. The engine is supported at two points as shown.
2. The weight of the components are neglected.
3. Transmission friction losses are neglected.

Analysis:

1. Consider first the transmission. This member receives torque T from the engine and delivers torque RT to the propeller shaft[1] (through a universal joint, not shown). The propeller shaft applies equal and opposite reaction torque RT to the transmission, as shown. For equilibrium, torque $RT - T$ *must* be applied *to* the transmission housing *by* the engine structure to which it is attached.
2. The engine receives torques T and $RT - T$ from the transmission (action–reaction principle). Moment RT must be applied by the frame (through the engine mounts), as shown.
3. The propeller shaft is in equilibrium under the action of equal and opposite torques applied at its two ends.

Comment: This simplified power train analysis gives an estimate of the component forces and moments.

[1] Neglecting transmission friction losses.

Sample Problem 2.4 Automotive Transmission Components

Figure 2.4*a* shows a simplified version of the transmission represented in Figure 2.3. The engine is delivering a torque $T = 3000$ lb · in. to the transmission, and the transmission is in low gear with a ratio $R = 2.778$. (For this problem consider R to be a torque ratio, T_{out}/T_{in}. To the degree that friction losses are present, the speed ratio, ω_{in}/ω_{out}, would have to be slightly greater.) The other three portions of the figure show the major parts of the transmission. Gear diameters are given in the figure. Input gear A rotates at engine speed and drives countershaft gear B. Countershaft gear C meshes with main shaft output gear D. (The construction of the main shaft is such that the input and output ends rotate about a common axis, but the two halves are free to rotate at different speeds.) The main shaft is supported in the housing by bearings I and II. Similarly, the countershaft is supported by bearings III and IV. Determine all loads acting on the components shown in Figures 2.4*b*, *c*, and *d*, thus representing them as free bodies in equilibrium. Suppose that the forces acting between mating gear teeth are tangential. (This amounts to

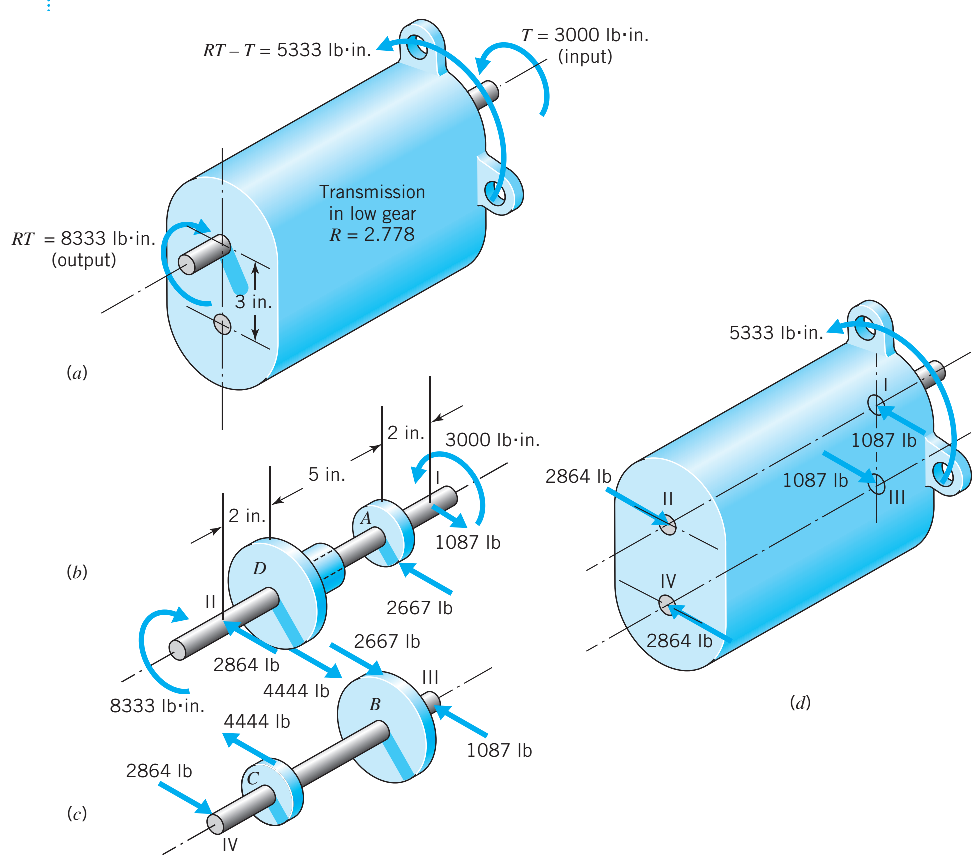


Figure 2.4
Free-body diagram of transmission and major components: (*a*) Complete transmission assembly. (*b*) Mainshaft (front and rear halves rotate freely with respect to each other). (*c*) Countershaft. (*d*) Housing. Note: Diameters of gears A and C are $2\frac{1}{4}$ in. Diameters of gears B and D are $3\frac{3}{4}$ in.

neglecting the radial and axial components of load. These load components are discussed in Chapters 15 and 16, dealing with gears.)

Solution

Known: A transmission of known general configuration and given ratio, $R = T_{out}/T_{in}$, receives a specified torque T from an engine. The arrangement and locations of the gears, shafts, and bearings inside the transmission are also known, as are the diameters of all gears.

Find: Determine all loads acting on the components.

Assumptions:

1. The forces acting between mating gear teeth are tangential. Radial gear tooth forces are ignored for simplicity.
2. The transmission input and output torques are steady (no acceleration or deceleration).

Analysis:

1. A very important initial observation is that equilibrium of the *total* transmission (Figure 2.4*a*) is *independent* of anything inside the housing. This free-body diagram would pertain equally well to transmissions of $R = 2.778$ having *no* gears inside—such as hydraulic or electric transmissions. In order for the transmission to work, whatever parts are inside *must* provide for the *torque of 5333 lb · in. to be reacted by the housing.* (A striking example of this concept came to the first author's attention when many persons sent the major automobile companies voluminous material pertaining to automatic transmission designs they wished to sell. To study all the drawings, analyses, descriptions, and so on would have required numerous hours. For many of the proposals, however, it could be quickly determined that there was no provision for a torque reaction to be transmitted to the housing, and that therefore the transmission could not possibly work.)
2. The input portion of the mainshaft (Figure 2.4*b*) requires the tangential force of 2667 lb to balance the input torque of 3000 lb · in., thus satisfying $\Sigma M = 0$ about the axis of rotation. This force is applied *to* gear *A* *by* gear *B*. Gear *A* applies an equal and opposite force to gear *B*, as shown in Figure 2.4*c*. Since there are no torques applied to the countershaft except by the two gears, it follows that gear *C* must receive a 4444-lb force from gear *D*. An opposite 4444-lb force is applied by gear *C* to gear *D*. Equilibrium of moments about the axis of the output half of the main shaft requires that a torque of 8333 lb · in. be applied to the output shaft by the propeller shaft as shown. (Note that the output torque can also be obtained by multiplying the input torque by the gear diameter ratios, *B*/*A* and *D*/*C*. Thus

$$3000 \text{ lb} \cdot \text{in.} \times \frac{3\frac{3}{4}\text{ in.}}{2\frac{1}{4}\text{ in.}} \times \frac{3\frac{3}{4}\text{ in.}}{2\frac{1}{4}\text{ in.}} = 8333 \text{ lb} \cdot \text{in.}$$

3. The force applied to the main shaft by bearing II is found by taking moments about bearing I. Thus

$$\Sigma M = 0: \quad (2667 \text{ lb})(2 \text{ in.}) - (4444 \text{ lb})(7 \text{ in.}) + (F_{\text{II}})(9 \text{ in.}) = 0$$

or

$$F_{\text{II}} = 2864 \text{ lb}$$

The force at bearing I is found by $\Sigma F = 0$ (or by $\Sigma M_{\text{II}} = 0$). Countershaft bearing reactions are found in the same way.

4. Figures 2.4*b* and *c* show bearing forces applied *to* the shafts, through the bearings, and *by* the housing. Figure 2.4*d* shows the corresponding forces applied *to* the housing, through the bearings, and *by* the shafts. The *only* members in contact with the housing are the four bearings and the bolts that connect it to the engine structure. Figure 2.4*d* shows that the housing is indeed a free body in equilibrium, as both forces and moments are in balance.

Comments:

1. The foregoing examples have illustrated how the powerful free-body-diagram method can be used to determine loads at various levels—that is, loads acting on a complex total device (as on an automobile), loads acting on a complex unit within the total device (as on the automotive transmission), and loads acting on one part of a complex unit (transmission countershaft).
2. Free-body-equilibrium concepts are equally effective and valuable in determining *internal* loads, as illustrated below in Sample Problem 2.5. This is also true of internal loads in components like the transmission countershaft in Figure 2.4*c*, as will be seen in the next section.

Sample Problem 2.5 Determination of Internal Loads

Two examples of load-carrying members are shown in Figures 2.5*a* and 2.6*a*. Using free-body diagrams, determine and show the loads existing at cross section *AA* of each member.

Solution

Known: The configuration and load orientation of two members is given.

Find: Determine and show the loads at cross section *AA* of each member.

Schematic and Given Data:

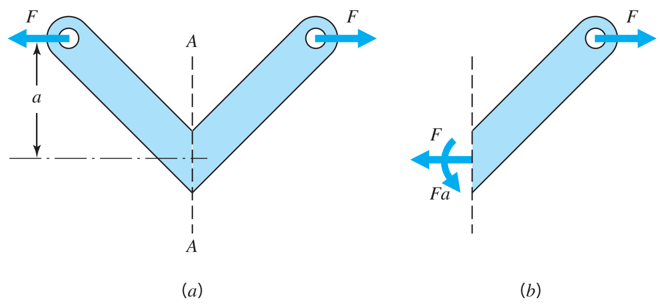


Figure 2.5
Loads acting on an internal section as determined from a free-body diagram.

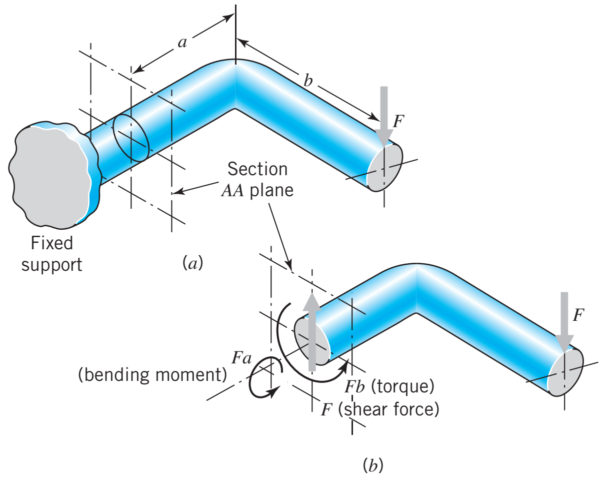


Figure 2.6
Loads acting on an internal section as determined from a free-body diagram.

Assumption: Deflections of the members do not cause a significant change in geometry.

Analysis: Figures 2.5*b* and 2.6*b* show segments on one side of section *AA* as free bodies in equilibrium. The forces and moments acting at the section are determined from the equations of equilibrium.

Comments:

1. Deflection of the member shown in Figure 2.5*a* would cause the moment aF to decrease. For most loads this change would be insignificant.
2. Please see Appendix G for a vectorial treatment of the load-carrying member shown in Figure 2.6.

The next two examples illustrate the determination of loads acting on three-force members where only one of the three forces is completely known and a second is known in direction only.

SAMPLE PROBLEM 2.6 Three-Force Member

Figure 2.7 shows a bell crank (link 2) that pivots freely with respect to the fixed frame (link 1). A horizontal rod (link 3, not shown) attaches at the top, exerting a force of 40 lb, as shown. (Note the subscript notation: F_{32} is a force applied *by* link 3 *to* link 2.) A rod 30° from horizontal (link 4, not shown) attaches to the bottom, exerting force F_{42} of unknown magnitude. Determine the magnitude of F_{42}, and also the direction and magnitude of force F_{12} (the force applied by fixed frame 1 to link 2 through the pinned connection near the center of the link).

SOLUTION

Known: A bell crank of specified geometry is loaded as shown in Figure 2.7.

Find: Determine F_{12} and the magnitude of F_{42}.

Schematic and Given Data:

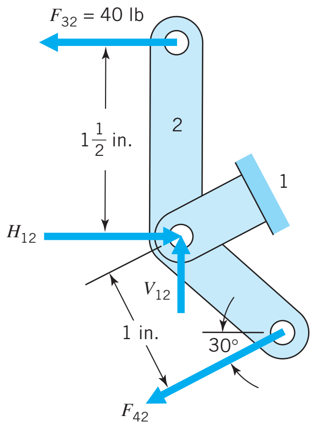


FIGURE 2.7
Bell crank forces—analytical solution.

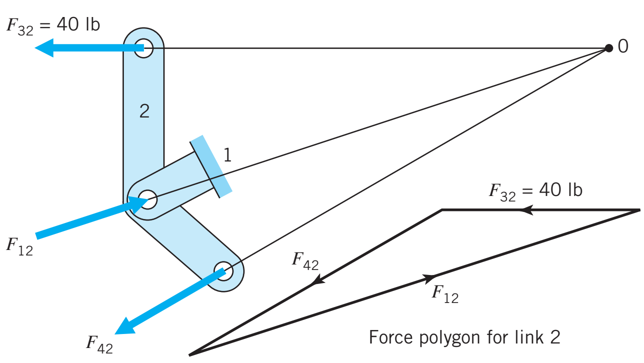


FIGURE 2.8
Bell crank forces—graphical solution.

Assumptions:

1. The pin joints are frictionless.
2. The bell crank is not accelerating.

Analysis A (Analytical):

1. Summation of moments about the pivot pin requires that $F_{42} = 60$ lb (note that 40 lb $\times$ $1\frac{1}{2}$ in. = 60 lb $\times$ 1 in.).
2. Dividing F_{12} into horizontal and vertical components, and setting the summation of vertical and horizontal forces acting on link 2 equal to zero yields $V_{12} = (60\text{ lb})(\sin 30°) = 30$ lb; $H_{12} = 40\text{ lb} + (60\text{ lb})\cos 30° = 92$ lb. The magnitude of F_{12} is $\sqrt{30^2 + 92^2} = 97$ lb; its direction is upward and to the right at an angle of $\tan^{-1} 30/92 = 18°$ from horizontal.

Analysis B (Graphical):

1. For equilibrium, the summation of moments of all forces acting on link 2 must be equal to zero when these moments are taken about *any* point, including point 0, which is the intersection of the two known force lines of action. Since two of the three forces have no moment about point 0, equilibrium requires that the third force also have no moment about 0. This can only be satisfied if the line of action of F_{12} also passes through 0.
2. We know one force completely, and the other two in direction only. A graphical solution for summation of forces equals zero is provided by the force polygon shown in Figure 2.8. This is constructed by first drawing known force F_{32} in proper direction and with length representing its 40-lb magnitude to any convenient scale. A line with the direction of F_{12} is drawn through either end of the vector representing F_{32}, and a line with the direction of F_{42} is drawn through the other end of this vector. Magnitudes of the two unknown forces can now be scaled from the polygon. (Note that the same result is obtained if a line of the direction of F_{42} is drawn through the *tail* of vector F_{32}, with the direction of F_{12} being drawn through the *tip* of F_{32}.)

Comment: The analytical solution solved the three equations for equilibrium in a plane for three unknowns. This same solution of simultaneous equations was accomplished graphically in Figure 2.8. An understanding of the graphical procedure adds to our insight into the nature of the force directions and magnitudes necessary for equilibrium of the link. Note that Figure 2.7 and Figure 2.8 show correct free body diagrams if the support link 1 is removed.

SAMPLE PROBLEM 2.7 Human Finger as Three-Force Member

The principles of mechanical engineering design that are traditionally applied to components of inanimate machines and structures are being increasingly applied in the relatively new field of *bioengineering*. A case in point is the application of free-body load analysis procedures to the internal load-carrying components of the human finger in studies of arthritic deformity [2,4]. Figure 2.9 illustrates one simplified portion of this study wherein a 10-lb pinch force at the tip of a finger is created by muscle contraction causing tendon force F_t. Determine the force in the tendon and in the finger bone.

SOLUTION

Known: The thumb and finger exert a known pinching force on a round object. The geometry is given.

Find: Estimate the tensile force in the tendon and the compressive force in the finger bone.

Schematic and Given Data:

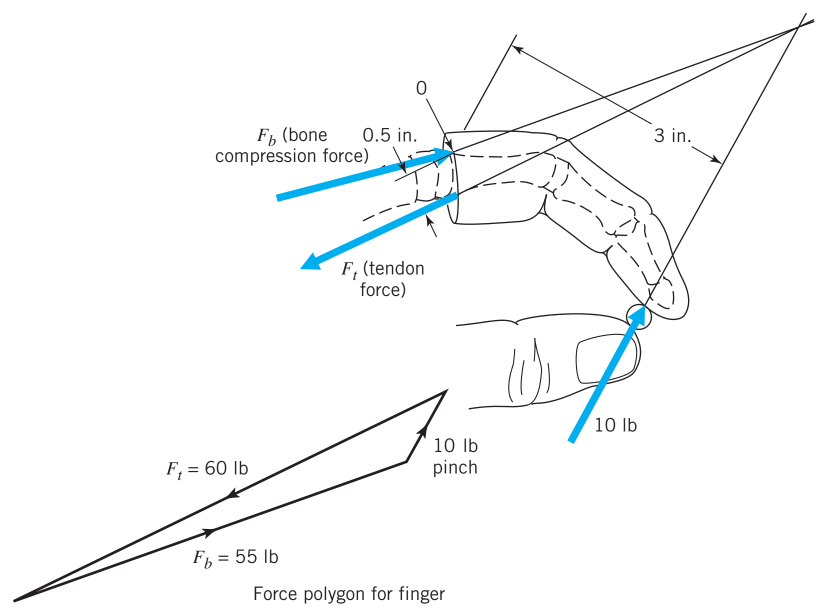


FIGURE 2.9
Force study of human finger.

Assumptions:

1. The load in the finger is carried solely by the tendon and the bone.
2. The finger is not accelerating.
3. The weight of the finger can be neglected.

Analysis: Since the tendon force has a moment arm about pivot point 0 of only one-sixth that of the pinch force, the tensile force in the tendon must be 60 lb. The force polygon shows that the compressive force between the finger bone (proximal phalanx) and mating bone in the hand (metacarpal) is about 55 lb—a value that may cause crushing of deteriorated arthritic bone tissue.

Other examples of free-body-diagram analysis are given in [1], [3], and [5].

2.3 Beam Loading

"Beam loading" refers to the lateral loading of members that are relatively long in comparison with their cross-sectional dimensions. Torsional or axial loading or both may or may not be involved as well. By way of review, two cases are shown in Figure 2.10. Note that each incorporates three basic diagrams: external loads, internal transverse shear forces (V), and internal bending moments (M). All expressions for magnitudes are the result of calculations the reader is advised to verify as a review exercise. (Reactions R_1 and R_2 are calculated first, on the basis of $\Sigma F = 0$ and $\Sigma M = 0$, with distributed load w treated as a concentrated load wb acting in the middle of span b.)

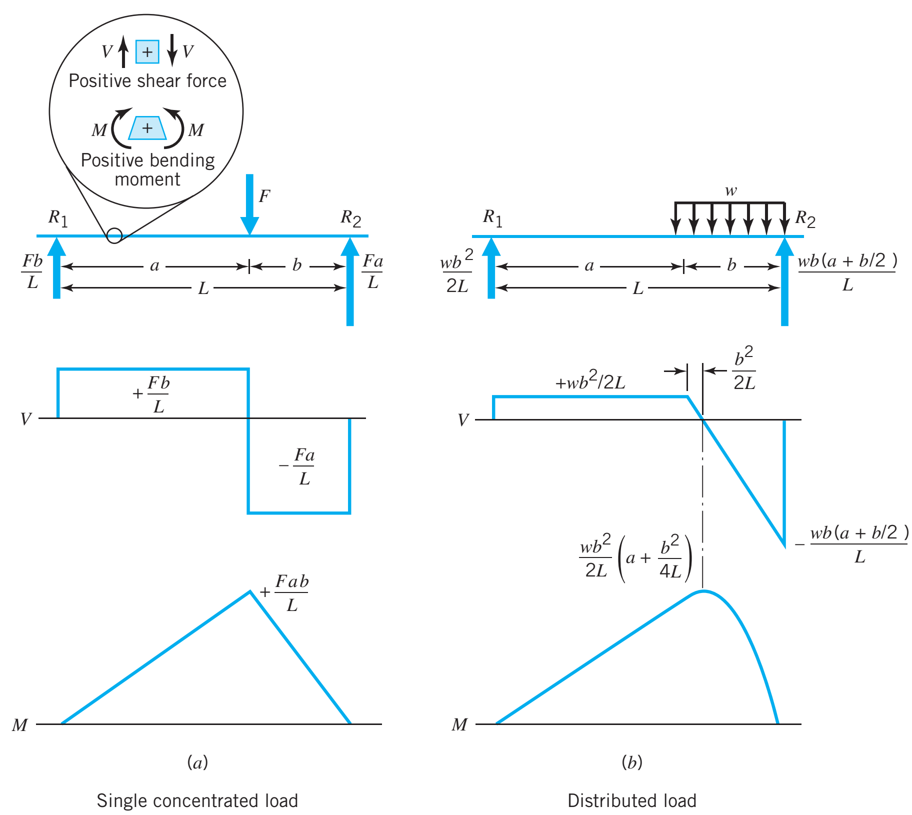


FIGURE 2.10
Examples of beam load, shear force, and bending moment diagrams.

The sign convention of the shear diagram is arbitrary, but the one used here is recommended: proceed from left to right, following the direction of the applied loads. In this case there are no loads to the left of reaction R_1 and hence no shear forces. At R_1 an upward force of Fb/L is encountered. Proceeding to the right, there are no loads—hence no *change* in the shear force—until the downward load of F is reached. At this point the shear diagram drops an amount F, and so forth. The diagram must come to zero at R_2, as no loads exist to the right of this reaction.

The internal transverse shear forces V and the internal bending moments M at a section of the beam are positive when they act as shown in Figure 2.10*a*. The shear at a section is positive when the portion of the beam to the left of the section tends to move upward with respect to the portion to the right of the section. The bending moment in a horizontal beam is positive at sections for which the top of the beam is in compression and the bottom is in tension. Generally, a positive moment will make the beam "smile."

The sign conventions presented are summarized as follows: The internal transverse shear forces V and the internal bending moments M at a section of beam are positive when they act in the directions shown in Figure 2.10*a*.

The (arbitrary) sign convention recommended here for bending follows from the relationship that

1. *The value of the shear force* (V) *at any point along the beam is equal to the slope of the bending moment diagram* (M) *at that point.*

Thus, a constant positive value of shear in the left portion of the beam results in a constant positive slope of the bending moment diagram over the same portion.

The student is reminded of three other important rules or relationships relative to load, shear, and moment diagrams.

2. *The value of local load intensity at any point along the beam is equal to the slope of the shear force diagram* (V) *at that point.* (For example, the end supports, acting as "points," produce a theoretically infinite upward force intensity. Hence, the slope of the shear diagram at these points is infinite.)

3. *The difference in the values of the shear load, at any two points along the beam, is equal to the area under the load diagram between these same two points.*

4. *The difference in the values of the bending moment, at any two points along the beam, is equal to the area under the shear diagram between these two points.*

Sample Problem 2.8 Internal Loads in a Transmission Countershaft

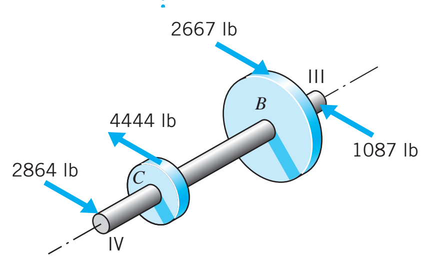
Locate the cross section of the shaft in Figure 2.11 (Figure 2.4*c*) that is subjected to the greatest loading, and determine the loading at this location.

Solution

Known: A shaft of uniform diameter and given length supports gears located at known positions B and C on the shaft.

Find: Determine the shaft cross section of greatest loading and the loads at this section.

Schematic and Given Data:



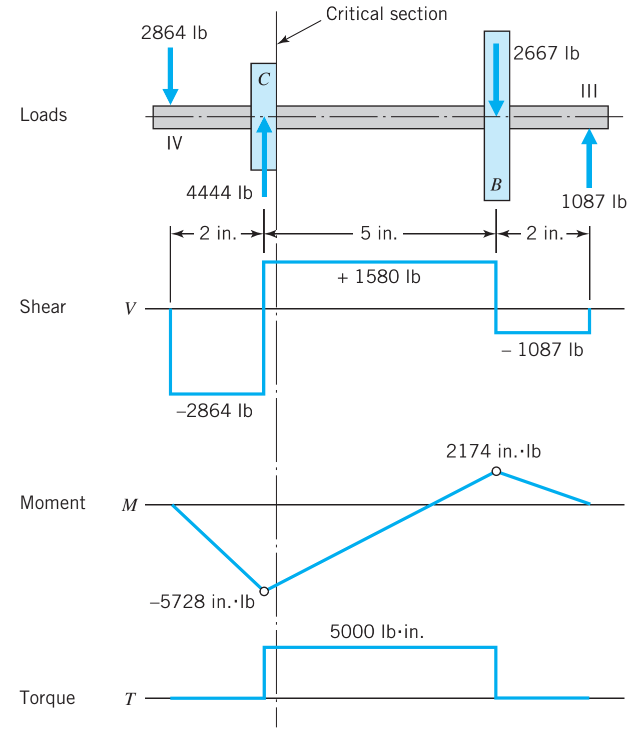


FIGURE 2.11
Countershaft loading, and determination of critical section.

Assumptions:

1. The shaft and gears rotate at uniform velocity.
2. Transverse shear stresses are negligible in comparison to bending and torsional shear stresses.

Analysis:

1. Figure 2.11 shows loading, shear, moment, and torque diagrams for this shaft. Note in particular the following.
   - **a.** The load diagram is in equilibrium—the forces and moments acting in the plane of the paper are balanced.
   - **b.** The recommended sign convention and the four basic relationships just given in italics are illustrated.
   - **c.** The sign convention used in the *torque diagram* is arbitrary. Zero torque exists outboard of the gears, for bearing friction would normally be neglected. Torques of (4444 lb)(2.25 in./2) and (2667 lb)(3.75 in./2) are applied to the shaft at C and B.
2. The critical location of the shaft is just to the right of gear C. Here we have maximum torque together with essentially maximum bending. (The transverse shear force is less than maximum, but except for highly unusual cases involving extremely short shafts, shear loads are unimportant in comparison with bending loads.)

Comment: Figure 2.12 shows the portion of the countershaft to the left of the critical section as a free body. Note that this *partial* member constitutes a *free body in equilibrium* under the action of all loads external to it. These include the external loads shown, and also the *internal* loads applied *to* the free body *by* the right-hand portion of the countershaft.

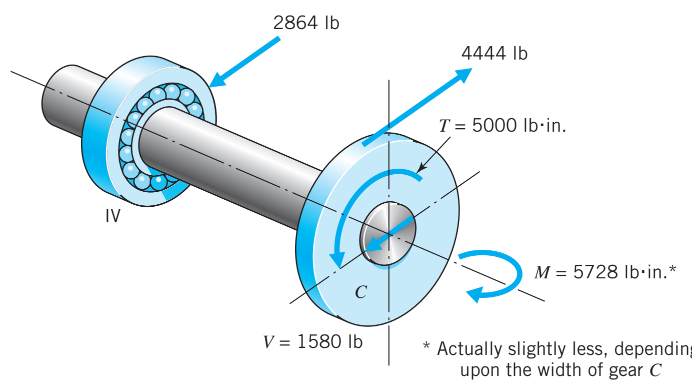


FIGURE 2.12
Loading at countershaft critical section.

2.4 Locating Critical Sections—Force Flow Concept

The sections chosen for load determination in the previous examples (i.e., in Figures 2.5, 2.6, and 2.12) were, by simple inspection, clearly those subjected to the most critical loading. In more complicated cases, however, several sections may be critical, and their locations less obvious. In such instances it is often helpful to employ an orderly procedure of following the "lines of force" (approximate paths taken by the force, determined by simple inspection) through the various parts, and noting along the way any sections suspected of being critical. Such a procedure is illustrated in the following example.

SAMPLE PROBLEM 2.9 Yoke Connection

Using the force flow concept, locate the critical sections and surfaces in the members shown in Figure 2.13.

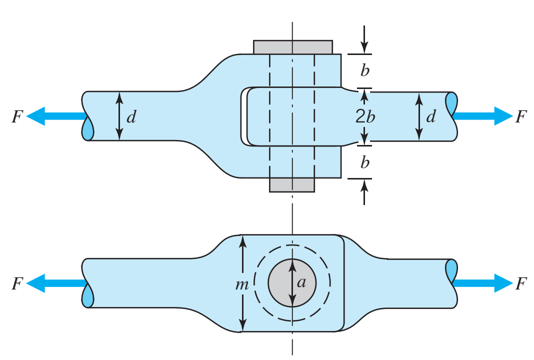


FIGURE 2.13
Yoke connection.

Solution

Known: A yoke connection is loaded in tension.

Find: Locate the critical sections and surfaces in the yoke fork, pin, and blade.

Schematic and Given Data:

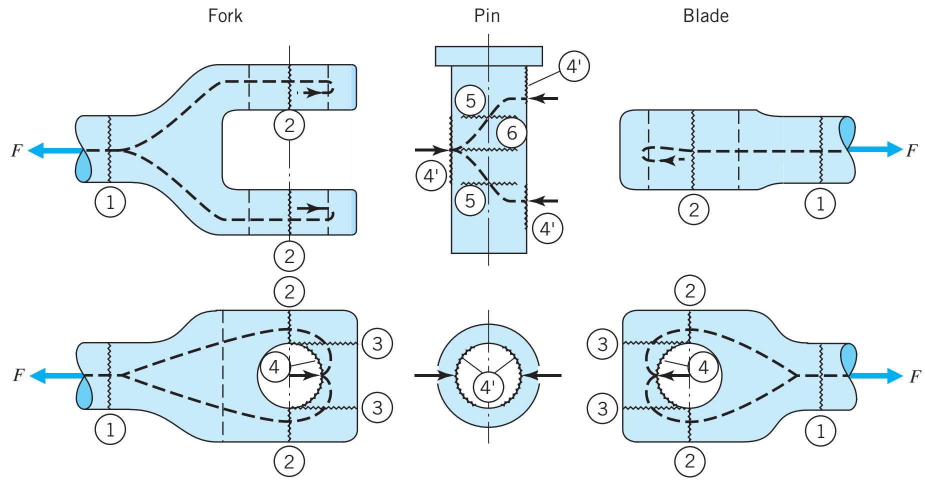


Figure 2.14
Force flow lines and critical sections in yoke connection.

Assumptions:

1. The weight of the yoke connection can be ignored.
2. The load is divided equally between the two prongs of the fork (the loads and yoke connection are perfectly symmetrical).
3. The load in each prong is divided equally between the portions on each side of the hole.
4. Distributed loads are represented as concentrated loads.
5. The effects of pin, blade, and fork deflections on load distribution are negligible.
6. The pin fits snugly in the fork and blade.

Analysis: A force flow path through each member is indicated by the dashed lines in Figure 2.14. Along this path from left to right, the major critical areas are indicated by the jagged lines and identified by the circled numbers.

a. Tensile loading exists at section ① of the fork. If the transition sections have ample material and generous radii, the next critical location is ②, where the force flow encounters a bottleneck because the area is reduced by the holes. Note that with this symmetrical design, force F is divided into four identical paths, each having an area at location ② of $\frac{1}{2}(m - a)b$.

b. The force flow proceeds on to the next questionable section, which is at ③. Here, the turning of the flow path is associated with shearing stresses tending to "push out" the end segments bounded by jagged lines ③.

c. The next critical area is interface ④ and ④′, where bearing loading exists between the fork-hole and pin surfaces, respectively. In like manner, equal bearing loads are developed at the interface between the pin and the blade-hole surfaces.

d. The forces at ④′ load the pin as a beam, causing direct shear loading of sections ⑤ (note that the pin is in "double shear" as the two surfaces ⑤ are loaded in parallel, each carrying a shear load of $F/2$). Moreover, the bearing loads produce a maximum bending moment at area ⑥, in the center of the pin.

e. After the forces emerge from the pin and enter the blade, they flow across critical areas ④, ③, ②, and ①, which correspond directly to the like-numbered sections of the fork.

f. Although not brought out in the simplified force-flow pattern of Figure 2.14, it should be noted that the bearing loads applied to the surfaces of the holes are not concentrated on the load axis but are, as assumed, *distributed* over these surfaces as shown in Figure 2.15. This gives rise to *hoop tension* (or circumferential tensile loading), tending to cause tensile failure in the section identified as ⑦.

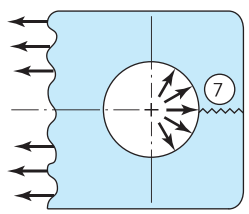


FIGURE 2.15
Distributed bearing loading may cause hoop tension failure at 7.

Comments: Although the determination of stresses at the various critical sections is beyond the scope of this chapter, this is a good time to give a word of caution regarding the simplifying assumptions that we might make when these stresses are calculated.

a. The section at ② might be assumed to be in uniform tension. Actually, bending is also present, which *adds* to the tension at the inner, or hole surface, and subtracts at the outer surface. This can be visualized by imagining the distortion of fork and blade members made of rubber and loaded through a metal connecting pin. The *quantitative* evaluation of this effect involves details of the geometry and is not a simple matter.

b. The distribution of compressive loading on surfaces ④ and ④′ might be assumed uniform, but this could be far from the actual case. The major factors involved are the fit of the pin in the hole and the rigidity of the members. For example, bending of the pin tends to cause highest bearing loading near the fork–blade interfaces. Moreover, the extent of pin bending depends not only on its own flexibility but also on the tightness of fit. The degree to which the pin is restrained from bending by a close fit has a major influence on the pin-bending stresses.

Like many engineering problems, this one illustrates three needs: (1) to be able to make reasonable simplifying assumptions and get usable answers quickly, (2) to be *aware* that such assumptions were made and interpret the results accordingly, and (3) to make a good engineering judgment whether a simplified solution is adequate in the particular situation, or whether a more sophisticated analysis, requiring more advanced analytical procedures and experimental studies, is justified.

2.5 Load Division Between Redundant Supports

A *redundant* support is one that could be removed and still leave the supported member in equilibrium. For example, in Figure 2.16, if the center support (which

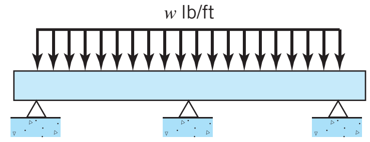


FIGURE 2.16
Redundant support.

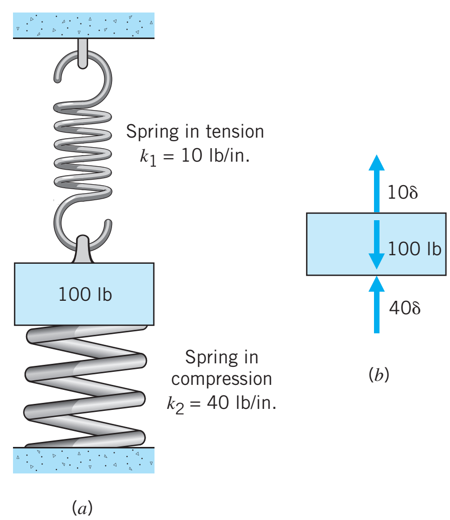


FIGURE 2.17
Weight supported redundantly from springs above and below.

is redundant) were removed, the beam would be held in equilibrium by the end supports. When redundant supports (reactions) are present, the simple equations of equilibrium no longer suffice to determine the magnitude of load carried by *any* of the supports. This is true because there are more unknowns than equilibrium equations.

A redundant support *adds stiffness* to the structure and carries a portion of the load proportional to its stiffness. For example, the 100-lb weight in Figure 2.17*a* is supported from below by a coil spring in compression and above by a coil spring in tension. (Assume that except for the 100-lb load, the springs would be stress-free). The top spring has a *rate* or *stiffness constant* of 10 lb/in., the lower spring a constant of 40 lb/in. Under the pull of gravity the weight moves downward, stretching the top spring by an amount δ and compressing the lower spring by the same amount. Thus, the weight is in equilibrium under the action of a gravity force of 100 lb and spring forces of 10δ and 40δ (Figure 2.17*b*). From $\Sigma F = 0$, we have $\delta = 2$ in. Hence, the top and bottom spring forces are 20 and 80 lb, respectively, illustrating that the *load is divided in proportion to the stiffness of the redundant supports.*

Suppose now that in order to reduce the deflection a third support is added—this in the form of a 10-in. length of steel wire, 0.020 in. in diameter inside the top spring, with the top end of the wire attached to the upper support and the lower end attached to the weight. The spring rate (AE/L) of the wire would be about 942 lb/in., and the *total* stiffness supporting the weight would then be 992 lb/in. Hence the wire would carry 942/992 of the 100-lb load, or 95 lb. Its stress would be

$$\frac{P}{A} = \frac{95 \text{ lb}}{0.000314 \text{ in.}^2}, \qquad \text{or} \quad 303{,}000 \text{ psi}$$

which is a value in excess of its probable tensile strength; hence it would fail (break or yield). Thus, *the strengths of redundant load-carrying members should be made approximately proportional to their stiffnesses.*

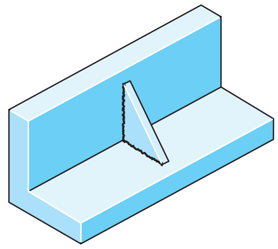


FIGURE 2.18
Web reinforcement added to an angle iron.

To illustrate this point further, consider the angle iron shown in Figure 2.18. Suppose that when installed as part of a machine or structure, the angle iron has inadequate rigidity in that the 90° angle deflects more than desired, although this does not cause breakage. The angular deflection is reduced by welding the small triangular web in place as shown. It becomes a redundant support that limits angular deflection. But it may well add stiffness far out of proportion to its strength. Cracks may appear in or near the welded joint, thereby eliminating the added stiffness. Furthermore, the cracks so formed may propagate through the main angle iron. If so, the addition of the triangular "reinforcement" would actually *weaken* the part. Sometimes failures in a complicated structural member (as a casting) can be corrected by *removing* stiff but weak portions (such as thin webs), *provided* the remaining portions of the part are sufficiently strong to carry the increased load imposed upon them by the increased deflection, and provided, of course, the increased deflection is itself acceptable.

A useful procedure (Castigliano's method) for calculating redundant load reactions for a completely *elastic* system is given in Section 5.9. At the other extreme, the loading pattern associated with *ductile* failure of a set of redundant supports is discussed in the next section.

2.6 Force Flow Concept Applied to Redundant Ductile Structures

As noted in Section 2.5, loads shared among parallel redundant paths are divided in proportion to path stiffnesses. If the paths are brittle and the load is increased to failure, one path will fracture first, thereby transferring its share of the load to the other paths, and so on, to failure of all paths. For the usual case involving materials of some ductility, one path will *yield* first, thereby reducing its stiffness (stiffness then being proportional to the *tangent modulus*),[2] which allows some of its load to be transferred to other paths. For the ductile case, general yielding of the total structure occurs only after the load has been increased sufficiently to bring all the parallel paths to their yield strengths.

The force flow concept, introduced in Section 2.4, is helpful in dealing with ductile redundant structures. This is illustrated in the following example.

SAMPLE PROBLEM 2.10 Riveted Joint [1]

Figure 2.19*a* shows a triple-riveted butt joint, wherein two plates are butted together and loads are transmitted across the joint by top and bottom straps. Each strap has a thickness of two-thirds the plate thickness and is made of the same material as the plates. Three rows of rivets attach each plate to the straps, as shown. The rivet pattern that is drawn for a width of one pitch is repeated over the full width of the joint. Determine the critical sections, and discuss the strength of the joint, using the force flow concept.

[2] The tangent modulus is defined as the slope of the stress–strain diagram at a particular stress level.

Solution

Known: A triple-riveted butt joint of specified geometry is loaded in tension.

Find: Determine the critical sections.

Schematic and Given Data:

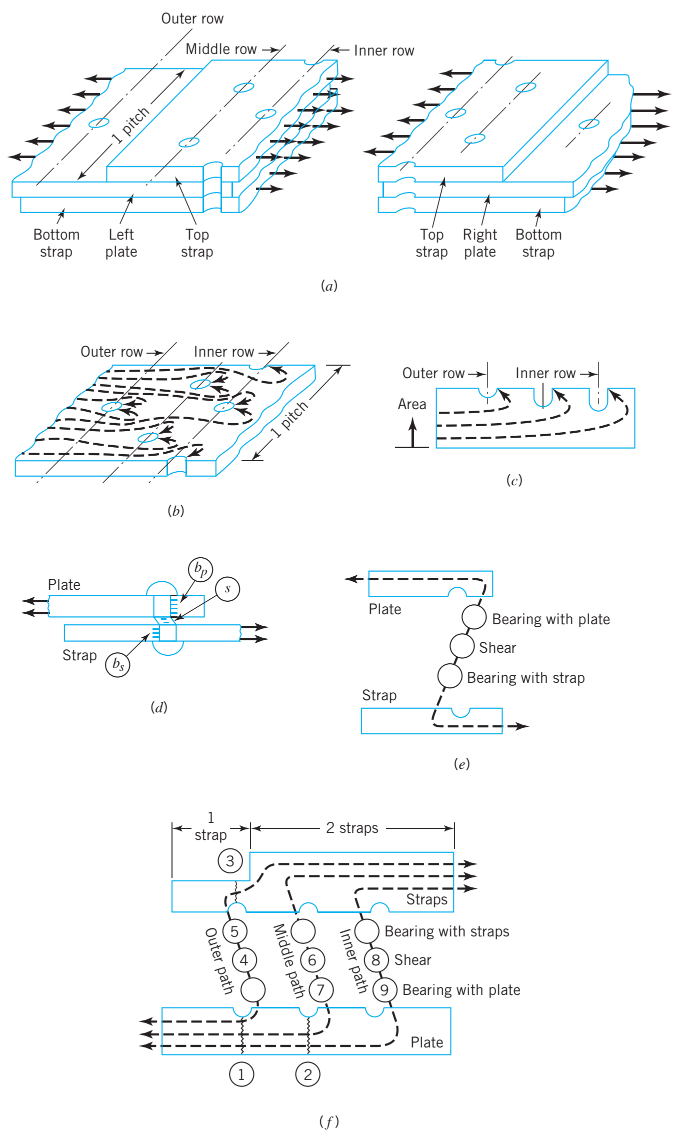


Figure 2.19

Force flow concept applied to triple-riveted butt joint. (*a*) Complete joint, broken at center, showing total load carried by straps. (*b*) Force flow through plate to rivets. (*c*) Diagram of force flow versus plate cross-sectional area. (*d*) Force flow through rivet. (*e*) Diagrammatic representation of force flow through rivet. (*f*) Complete diagrammatic representation of force flow.

Assumptions:

1. The weight of the riveted joint can be ignored.
2. The load is distributed evenly across the width of the joint (there is no misalignment).
3. The rivets fit snugly in the plate and straps.

Analysis:

1. Figure 2.19*b* shows a diagrammatic sketch of the force flow pattern in the plate. A portion of the load is transferred to each of the three rows of rivets. (Since static equilibrium could be satisfied by using any one row, the structure is redundant.) Note that at the outer row the entire force flows across a section containing one rivet hole. At the middle row, a section containing two rivet holes is subjected to all the force not going to the outer row of rivets. The section of plate at the inner row transmits only the force going to the inner row of rivets. This relation between force and area at each section of plate is represented diagrammatically in Figure 2.19*c*. Note the representation of the reduction in area owing to rivet holes at the middle and inner rows being twice those at the outer row.
2. Figure 2.19*d* shows how each rivet is associated with three important loadings: bearing with the plate, shear, and bearing with the strap. A diagrammatic force flow representation of this is given in Figure 2.19*e*, which shows the force path encountering five critical sections in series: the reduced tensile area containing the holes in the plate, the sections corresponding to the three loadings involving the rivet, and the reduced tensile area of the strap.
3. Figure 2.19*f* shows a similar representation of the entire joint. Critical sections are identified and numbered ① to ⑨. Basically, three parallel force paths are involved, one going through each row of rivets. Starting at the lower left, all three paths flow across the reduced plate section at the outer row. This is critical area ①. Failure at this point severs all force flow paths, causing total fracture of the joint. Only two paths flow across the plate at ②, but since the area here is less than at ①, failure is possible. Failure is not possible in the plate at the inner path, for only one force path flows across an area identical with that at ②.
4. Turning to the possibilities of tensile failure in the straps, note that the relative thickness of plate and straps is such that strap tensile failure is possible only at the outer row at critical point ③.
5. With respect to possibilities of failure involving the rivets themselves, each rivet is vulnerable in shear and at *one* bearing area (whichever is smaller). In the outer row, the vulnerable bearing area involves the strap, because the strap is thinner than the plate. In the other rows, the vulnerable bearing area involves the plate, for the strap-bearing load is shared by two straps, the combined thickness of which exceeds that of the plate. Note also that the middle- and inner-row rivets divide their shear load between *two* areas (i.e., they are loaded in *double shear*), whereas the outer-row rivets have but a single shear area. The possibilities of rivet failure are numbered ④ to ⑨.

6. The distribution of load among the three redundant paths depends on relative stiffnesses; but since riveted joints are invariably made of ductile materials, slight local yielding permits a redistribution of the load. Thus, final failure of the joint will occur only when the external load exceeds the combined load-carrying capacity of all three paths. This involves simultaneous failure of all paths and can occur in three possible ways:

a. Tensile failure at ①.

b. Simultaneous failure of the weakest link in each of the three paths.

c. Simultaneous failure at ② and at the weakest link in the outer path (③, ④, or ⑤).

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1. Juvinall, Robert C., *Engineering Consideration of Stress, Strain and Strength,* McGraw-Hill, New York, 1967.
2. Juvinall, Robert C., "An Engineering View of Musculoskeletal Deformities," Proceedings of the IVth International Congress of Physical Medicine, Paris, 1964.
3. Riley, William F., L. D. Sturges, and D. H. Morris, *Statics and Mechanics of Materials: An Integrated Approach,* 2nd ed., Wiley, New York, 2001.
4. Smith, Edwin M., R. C. Juvinall, L. F. Bender, and J. R. Pearson, "Flexor Forces and Rheumatoid Metacarpophalangeal Deformity," *J. Amer. Med. Assoc.,* **198** (Oct. 10, 1966).
5. Craig, R. R., Jr., *Mechanics of Materials,* 3rd ed., Wiley, New York, 2011.

Problems

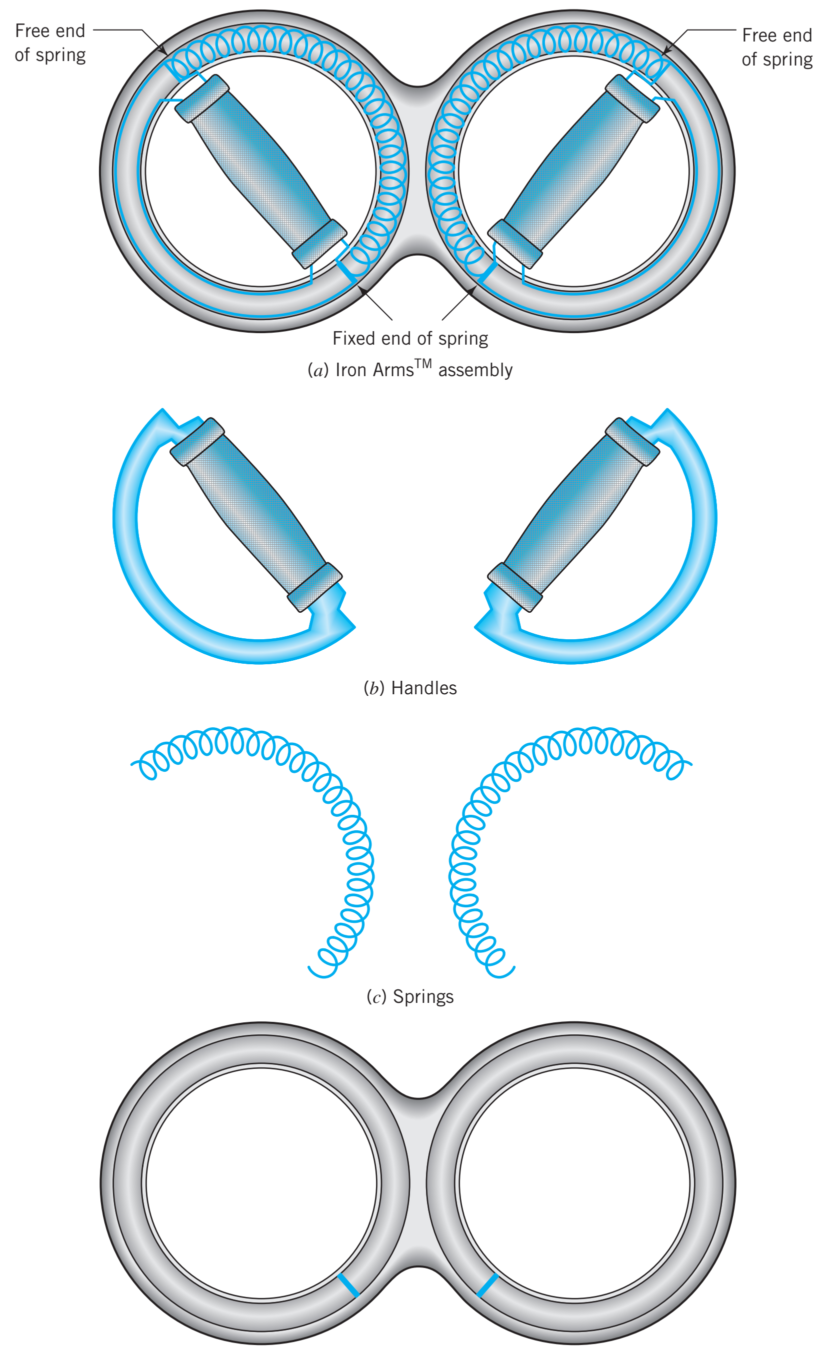
Section 2.2

2.1D Write definitions for the terms *free-body diagram, equilibrium analysis, internal loads, external loads,* and *three-force members.*

2.2 The Iron Arms™ rotating forearm grips exercise the forearms by resisting rotation of the handle grips—see Figure P2.2. The rotation of the handle about its center (the center of the ring) is opposed by the force on the free end of the helical compression spring. The handle is shaped like a "D." The curved portion of the D slides smoothly inside the hollow ring of the housing. The straight portion of the D is for hand gripping and has foam padding.

The handle grip length is approximately 4.0 in. long and 1.25 in. in diameter. The rings have an outer diameter of 7.75 in. and an inner diameter of 5.375 in. The entire Iron Arm assembly has an overall length of 15.40 in., a width of 7.75 in. (outer ring diameter), and a thickness of 1.25 in.

When the right handle is rotated counterclockwise the spring is compressed and it resists handle rotation. Simultaneously, the left handle can be rotated clockwise, and the spring on the left will be compressed.



(*a*) Iron Arms™ assembly

(*b*) Handles

(*c*) Springs

(*d*) Housing (rings)

Figure P2.2

In the position shown in Figure P2.2 (a), the springs are both compressed (preloaded) and in static equilibrium. For this position, draw a free-body diagram of each spring, each handle, the housing (rings) and the Iron Arms assembly. Assume that there are no gravitational forces, frictional forces, or handle torque acting on the components.

2.3 Referring to the Iron Arms assembly as described in Problem 2.2, draw free-body diagrams for the springs, handles, housing (rings), and the Iron Arms assembly for the condition where a person has rotated the right handle 90º counterclockwise and the left handle 90º clockwise with their hands—see Figure P2.3. Assume that the assembly is in static equilibrium and that a pure torque is applied by hand to each handle grip. Neglect the weight of the components.

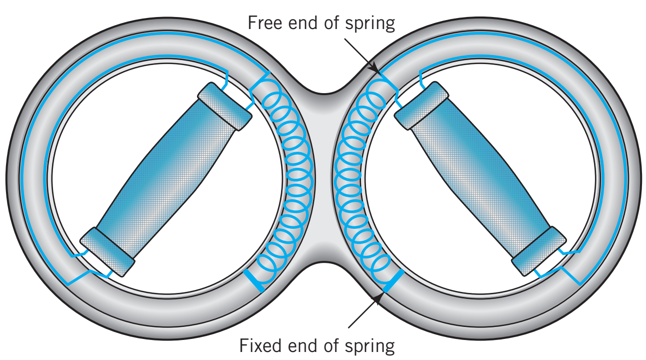


FIGURE P2.3

2.4 (a) If one person on each end of a thin, steel cable pulls with a force of 75 lb, what tension would exist in the cable? (b) If the cable at one end is attached directly and permanently to a tree and a person pulls on the other end with a force of 75 lb, what force would exist in the steel cable?

2.5D What forces act on a person walking on a level roadway? If that person walks on a level belt of a treadmill, what forces act on that person? Which activity takes more effort?

2.6D Draw a free-body diagram for the motorcycle of weight W shown in Figure P2.6D for (a) rear wheel braking only, (b) front wheel braking only, and (c) front and rear wheel braking. Also, determine the magnitudes of the forces exerted by the roadway on the two tires during braking for the above cases. The motorcycle has a wheel base of length L. The center of gravity is a distance c forward of the rear axle and a distance of h above the road. The coefficient of friction between the pavement and the tires is μ.

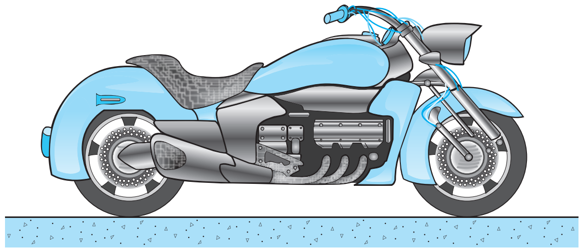


FIGURE P2.6D

2.7 Referring to Problem 2.6D, for $W = 1000$ lb, $L = 70$ in., $h = 24$ in., $c = 38$ in., and $\mu = 0.7$, determine the forces on the rear wheel for (a) and (b).

2.8 Draw a free-body diagram of an automobile of weight W that has a wheel base of length L during four-wheel braking. The center of gravity is a distance c forward of the rear axle and a distance of h above the ground. The coefficient of friction between the pavement and the tires is μ. Also show that the load carried by the two front tires during braking with the motor disconnected is equal to $W(c + \mu h)/L$.

2.9 Referring to Problem 2.8, for $W = 4000$ lb, $L = 117$ in., $c = 65$ in., and $h = 17.5$ in., and $\mu = 0.7$, determine the force on each of the two rear tires.

2.10 Repeat Problem 2.8, except assume that the automobile is towing a one-axle trailer of weight W_t. Determine the minimum stopping distance for the automobile and trailer assuming (a) no braking on the trailer and (b) full braking on the trailer. What is the minimum stopping distance for the automobile if it is not towing a trailer?

2.11 Repeat Problem 2.8, except assume that the automobile is traveling downhill at a grade of 10 : 1.

2.12D Select a metal with known density for solid rods A and B. Rod A and rod B are positioned inside a vertical wall channel C. Sketch free-body diagrams for rod A, rod B, and channel C, shown in Figure P2.12D. Also determine the magnitude of the forces acting on rod A, rod B, and channel C.

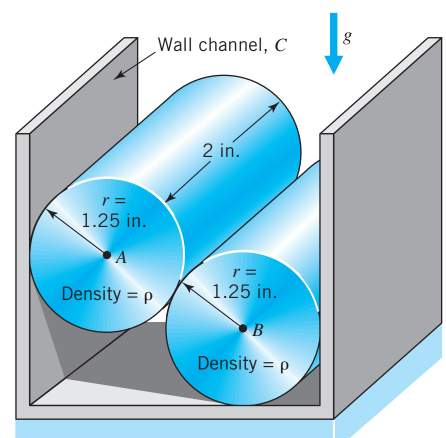


FIGURE P2.12D

2.13D Sketch free-body diagrams for sphere A, sphere B, and the container, shown in Figure P2.13D. Also determine the magnitude of the forces acting on sphere A, sphere B, and the container.

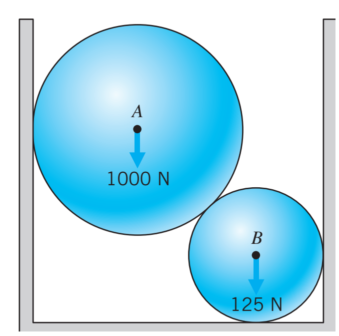


FIGURE P2.13D

2.14 Draw the free-body diagram for the pinned assembly shown in Figure P2.14. Find the magnitude of the forces acting on each member of the assembly.

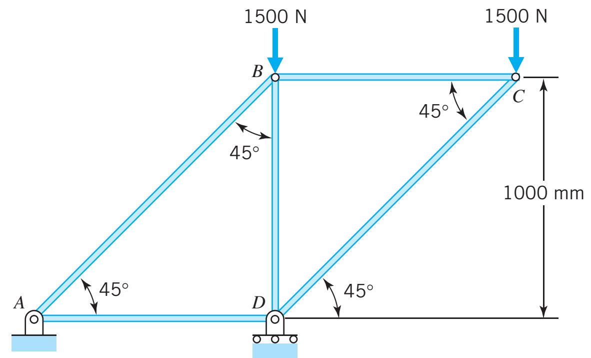


FIGURE P2.14

2.15 Referring to Problem 2.14, where the two 1500 N external forces are each reduced to 750 N, determine the forces on the structure at *A* and *D*.

2.16 The drawing (Figure P2.16) shows an exploded view of an 1800-rpm motor, a gear box, and a 6000-rpm blower. The gear box weighs 20 lb, with center of gravity midway between the two mountings. All shafts rotate counterclockwise, viewed from the blower. Neglecting friction losses, determine all loads acting on the gear box when the motor output is 1 hp. Sketch the gear box as a free body in equilibrium.

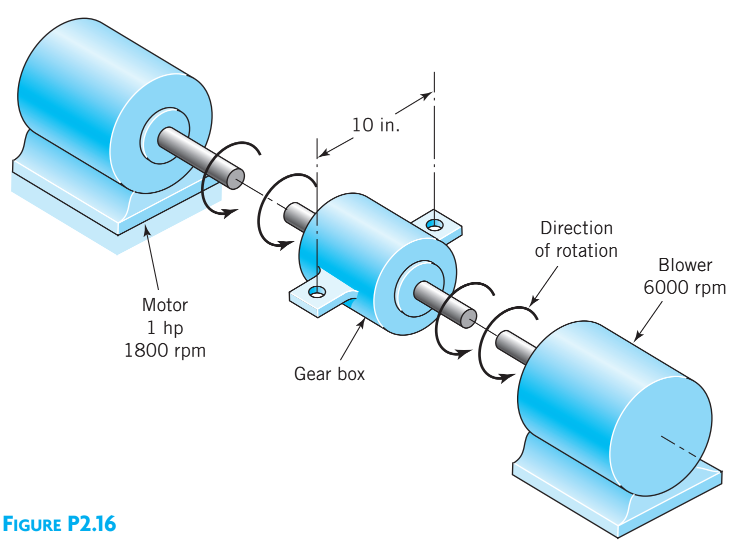


FIGURE P2.16

2.17 Repeat Problem 2.16 where the weight of the gearbox is 40 lb.

2.18 The motor shown operates at constant speed and develops a torque of 100 lb-in. during normal operation. Attached to the motor shaft is a gear reducer of ratio 5:1, i.e., the reducer output shaft rotates in the same direction as the motor but at one-fifth motor speed. Rotation of the reducer housing is prevented by the "torque arm," pin-connected at each end as shown in Figure P2.18. The reducer output shaft drives the load through a flexible coupling. Neglecting gravity and friction, what loads are applied to (a) the torque arm, (b) the motor output shaft, and (c) the reducer output shaft?

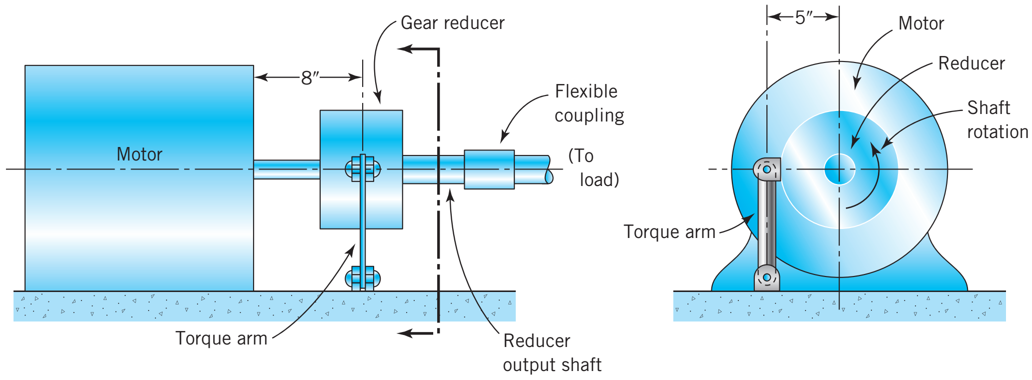


FIGURE P2.18

2.19 Repeat Problem 2.18 where the motor develops a torque of 200 lb-in. during operation.

2.20 The drawing (Figure P2.20) shows the engine, transmission, and propeller shaft of a prototype automobile. The transmission and engine are not bolted together but are attached separately to the frame. The transmission weighs 100 lb, receives an engine torque of 100 lb-ft at A through a flexible coupling, and attaches to the propeller shaft at B through a universal joint. The transmission is bolted to the frame at C and D. If the transmission ratio is -3, i.e., reverse gear with propeller shaft speed $= -\frac{1}{3}$ engine speed, show the transmission as a free body in equilibrium.

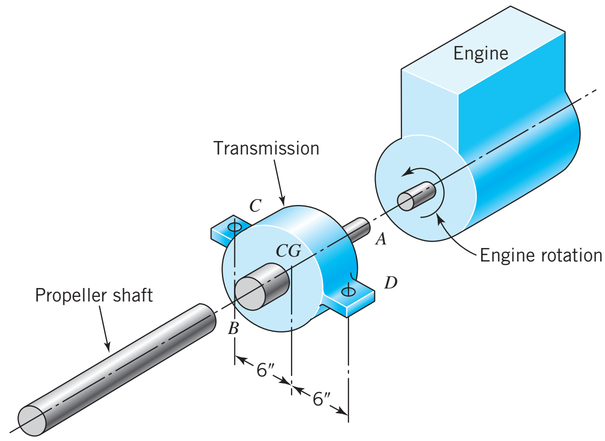


FIGURE P2.20

2.21 Repeat Problem 2.20 if the transmission weighs 50 lb.

2.22 The drawing (Figure P2.22) shows an electric fan supported by mountings at A and B. The motor delivers a torque of 2 N·m to the fan blades. They, in turn, push the air forward with a force of 20 N. Neglecting gravity forces, determine all loads acting on the fan (complete assembly). Sketch it as a free body in equilibrium.

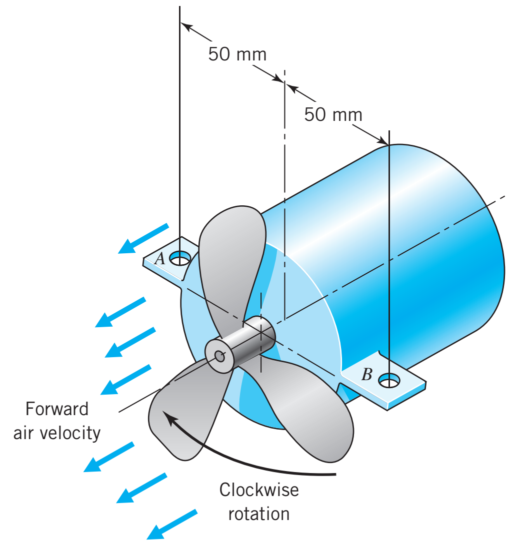


FIGURE P2.22

2.23 Repeat Problem 2.22 if the motor delivers a torque of 4 N · m to the fan blades and the blades push the air forward with a force of 40N.

2.24 Figure P2.24 shows an exploded drawing of a pump driven by a 1.5-kW, 1800-rpm motor integrally attached to a 4 : 1 ratio gear reducer. Reducer shaft C is connected directly to pump shaft C' through a flexible coupling (not shown). Face A of the reducer housing is bolted to flange A' of the connecting tube (a one-piece solid unit). Pump face B is similarly attached to flange B'. Sketch the connecting tube and show all loads acting on it. (Neglect gravity.)

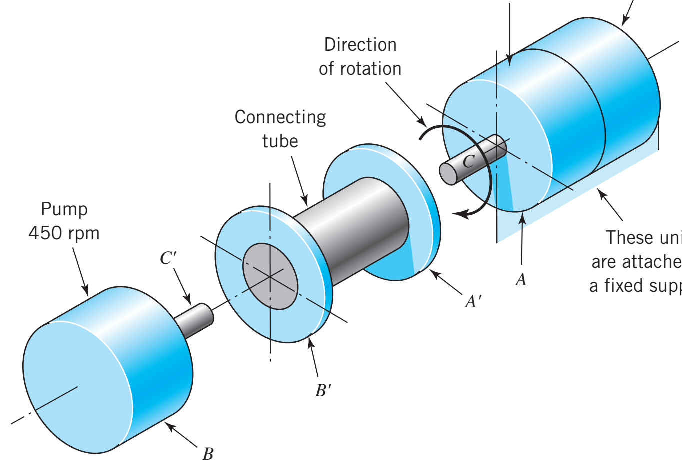


FIGURE P2.24

2.25 Referring to Problem 2.24. calculate the output torque if the 4:1 ratio gear reducer has an efficiency of 95%.

2.26 Figure P2.26 shows an exploded view of an airplane engine, reduction gear, and propeller. The engine and propeller rotate clockwise, viewed from the propeller end. The reduction gear housing is bolted to the engine housing through the bolt holes shown. Neglect friction losses in the reduction gear. When the engine develops 150 hp at 3600 rpm,

(a) What is the direction and magnitude of the torque applied *to* the engine housing *by* the reduction gear housing?

(b) What is the magnitude and direction of the torque reaction tending to rotate (roll) the aircraft?

(c) What is an advantage of using opposite-rotating engines with twin-engine propeller-driven aircraft?

[Ans.: (a) 109 lb · ft ccw, (b) 328 lb · ft ccw]

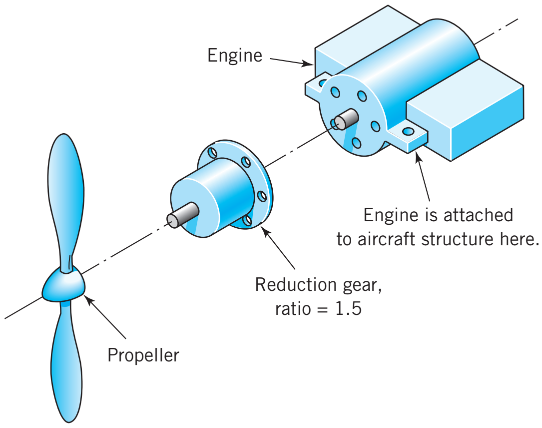


FIGURE P2.26

2.27 A marine engine delivers a torque of 200 lb-ft to a gearbox shown in Figure P2.27 which provides a reverse ratio of −4 : 1. What torque is required to hold the gearbox in place?

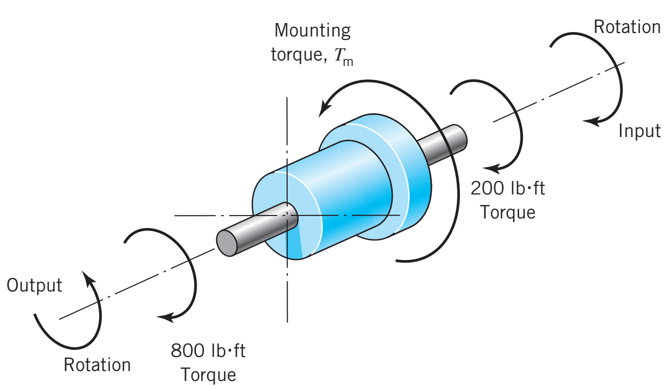


FIGURE P2.27

2.28 Repeat Problem 2.27 where the marine engine delivers a torque of 400 lb-ft.

2.29 A motor delivers 50 lb-ft torque at 2000 rpm to an attached gear reducer. The reducer and motor housings are connected together by six bolts located on a 12-in.-dia. circle, centered about the shaft. The reducer has a 4 : 1 ratio. Neglecting friction and weight, what average shearing force is carried by each bolt?

2.30D Select a single-cylinder reciprocating compressor. Sketch the crankshaft, connecting rod, piston, and frame as free bodies when the piston is 60° before head-end dead center on the compression stroke. Sketch the entire compressor as a single free body for this condition.

2.31 Figure P2.31 shows the gear reduction unit and propeller of an outboard motor boat. It is attached to the boat structure at the mounting flange at the top. The motor is mounted above this unit, and turns the vertical shaft with a torque of 20 N · m. By means of bevel gearing, this shaft turns the propeller at half the vertical shaft speed. The propeller provides a thrust of 400 N to drive the boat forward. Neglecting gravity and friction, show all external loads acting on the assembly shown. (Make a sketch, and show moments applied to the mounting flange using the notation suggested in the drawing.)

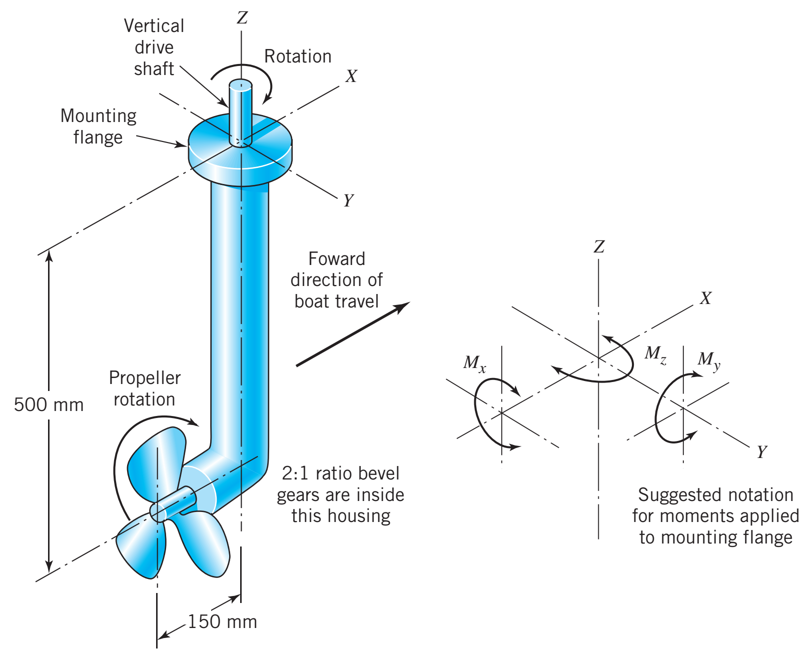


FIGURE P2.31

2.32 The drawing (Figure P2.32) represents a bicycle with an 800-N rider applying full weight to one pedal. Treat this as a two-dimensional problem, with all components in the plane of the paper. Draw as free bodies in equilibrium

(a) The pedal, crank, and pedal sprocket assembly.

(b) The rear wheel and sprocket assembly.

(c) The front wheel.

(d) The entire bicycle and rider assembly.

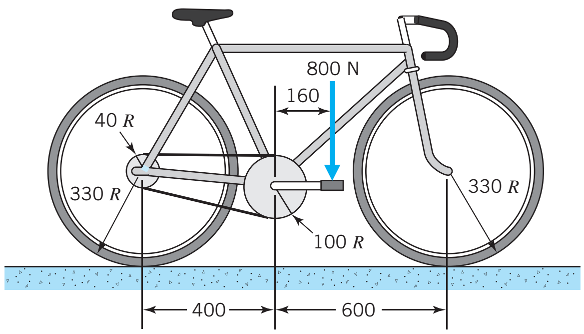


FIGURE P2.32

2.33 Referring to the bicycle in Problem 2.32, with a small rider applying her full weight of 400 N to one pedal, determine the forces on the rear and front tires.

2.34 The solid, continuous round bar shown in Figure P2.34 can be viewed as composed of a straight segment and a curved segment. Draw free-body diagrams for the segments 1 and 2. Also, determine the forces and moments acting on the ends of both segments. Neglect the weight of the member.

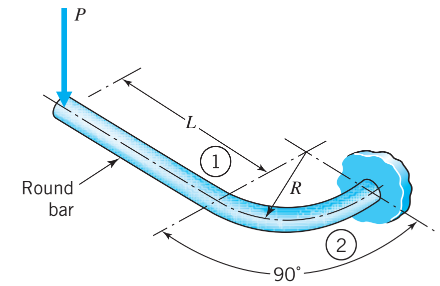


FIGURE P2.34

2.35 For the spring clip (Figure P2.35) having a force *P* acting on the free end, draw free-body diagrams for segments 1 and 2. Also, determine the force and moments acting on the ends of both segments. Neglect the weight of the member.

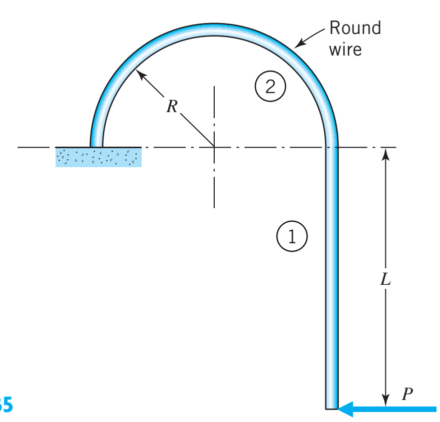


FIGURE P2.35

2.36 A semicircular bar of rectangular cross section has one pinned end (Figure P2.36). The free end is loaded as shown. Draw a free-body diagram for the entire semicircular bar and for a left portion of the bar. Discuss what influence the weight of the semicircular bar has on this problem.

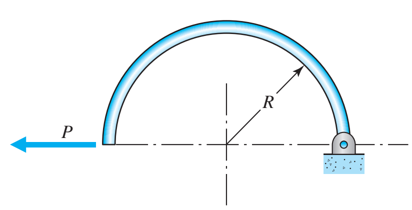


FIGURE P2.36

2.37 The drawing (Figure P2.37) shows a bevel gear reducer driven by an 1800-rpm motor delivering a torque of 12 N·m. The output drives a 600-rpm load. The reducer is held in place by vertical forces applied at mountings *A*, *B*, *C*, and *D*.

Torque reaction about the motor shaft is reacted at *A* and *B*; torque reaction about the output shaft is reacted at *C* and *D*. Determine the forces applied to the reducer at each of the mountings,

(a) Assuming 100% reducer efficiency.

(b) Assuming 95% reducer efficiency.

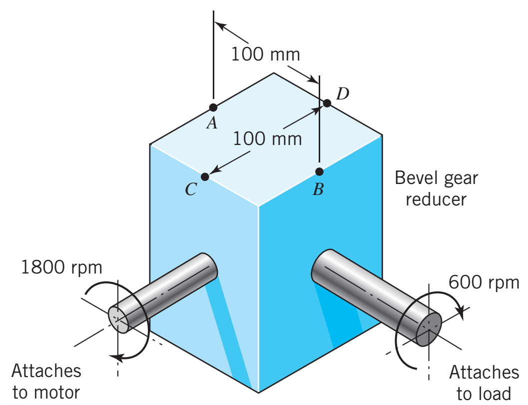


FIGURE P2.37

2.38 Repeat Problem 2.37 for a motor delivering a torque of 24 N·m at 1800 rpm.

2.39 Repeat Problem 2.37 with the distances $AB = 50$ mm and $CD = 50$ mm.

2.40 The drawings (Figure P2.40) pertain to a spur gear reducer. A motor applies a torque of 200 lb · ft to the pinion shaft, as shown. The gear shaft drives the output load. Both shafts are connected with flexible couplings (which transmit only torque). The gears are mounted on their shafts midway between the bearings. The reducer is supported by four identical mountings on the side of the housing, symmetrically spaced on 6-in. and 8-in. centers, as shown. For simplicity, neglect gravity and assume that forces between the gears (i.e., between gear and pinion) act tangentially. Sketch, as free bodies in equilibrium,

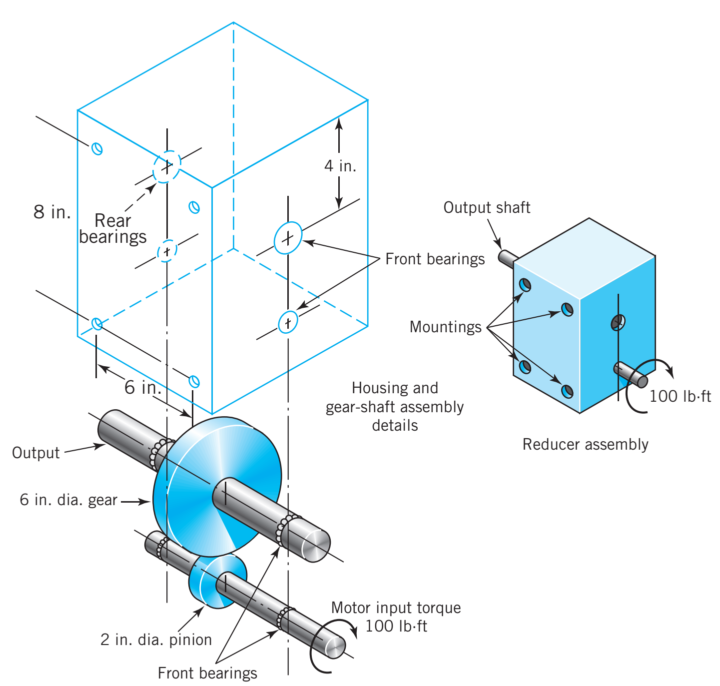


FIGURE P2.40

(a) The pinion and shaft assembly.
(b) The gear and shaft assembly.
(c) The housing.
(d) The entire reducer assembly.

2.41 A rim and hub are connected by spokes (springs) as shown in Figure P2.41. The spokes are each tightened to a tension of 20 lb. Draw a free-body diagram of (a) the hub, (b) the rim, (c) one spring, and (d) one-half (180°) of the rim. Ignore the weight of each component.

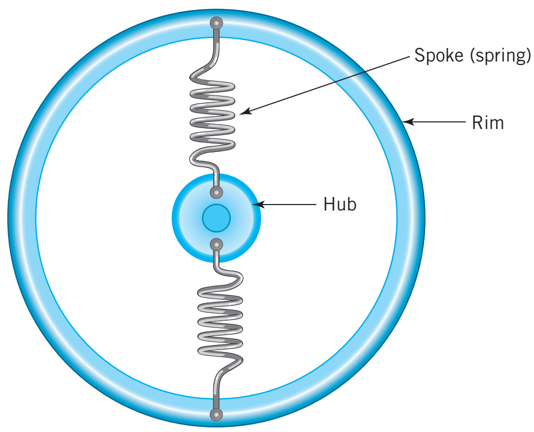


FIGURE P2.41

2.42 The drawing (Figure P2.42) is a highly simplified diagrammatic representation of the engine, transmission, drive shafts, and front axle of a four-wheel-drive car. All members shown may be treated as a single free body supported by mountings at *A*, *B*, *C*, and *D*. The engine rotates 2400 rpm and delivers a torque of 100 lb · ft. The transmission ratio is 2.0 (drive shafts rotate 1200 rpm); the front and rear axle ratios are both 3.0 (wheels

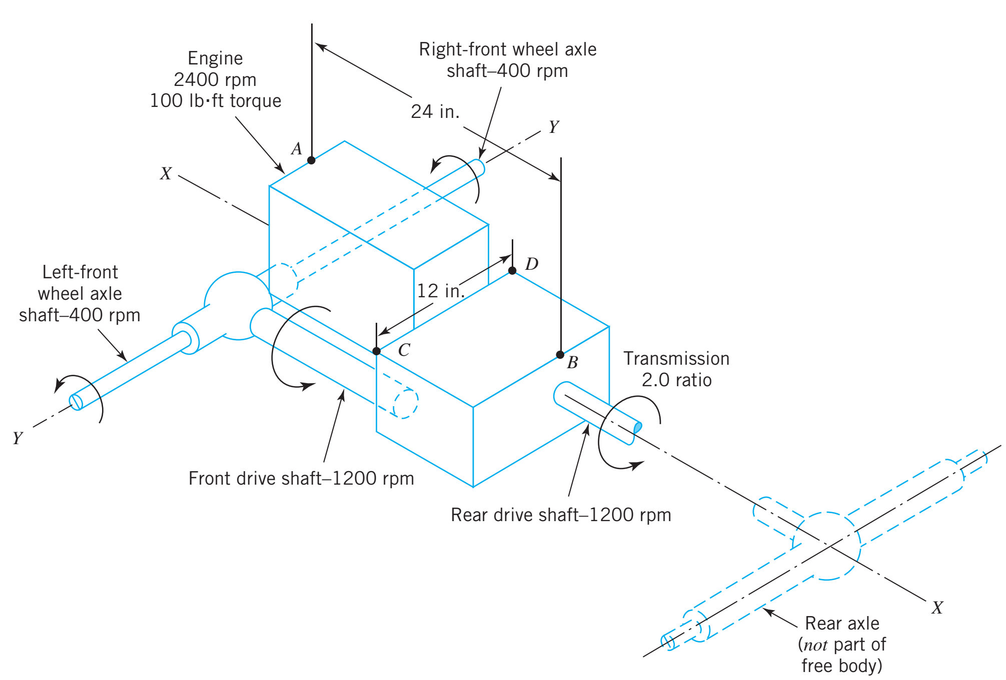


FIGURE P2.42

rotate 400 rpm). Neglect friction and gravity, and assume that the mountings exert only vertical forces. Determine the forces applied to the free body at *A*, *B*, *C*, and *D*.

[Ans.: 150 lb down, 150 lb up, 100 lb down, and 100 lb up, respectively]

2.43D The drawing in Figure P2.43D shows a mixer supported by symmetric mountings at *A* and *B*. Select a motor torque between 20 N · m and 50 N · m for driving the mixing paddles and then determine all loads acting on the mixer. Sketch the free body in equilibrium.

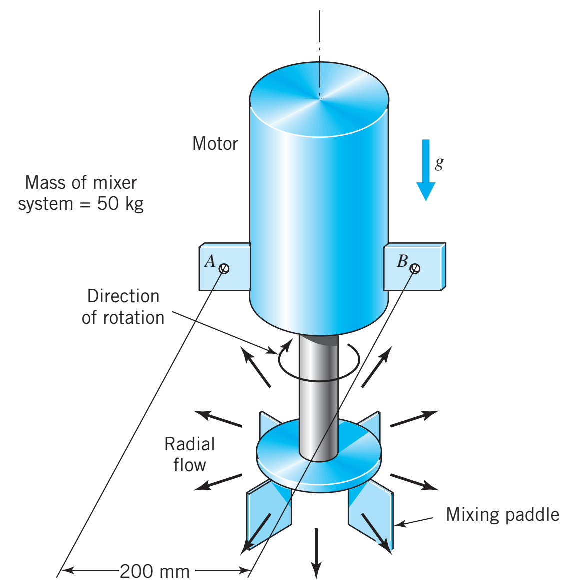


FIGURE P2.43D

2.44 Draw a free-body diagram of a rear-wheel-driven vehicle traveling at a steady speed on a straight and level roadway where the opposing forces are the (i) drag force, F_d, imposed on a vehicle by the surrounding air, the (ii) rolling resistance force on the tires, F_r, opposing the motion of the vehicle, and (iii) the forces of the road acting on the tires. Assume that the vehicle has a weight *W*. Also, describe how the free-body diagram changes if the accelerator pedal is pushed and the vehicle starts accelerating.

2.45 Repeat Problem 2.44 for a front-wheel-driven vehicle.

2.46 Draw a free-body-diagram for a large size binder clip for the position where the clip is held open and being readied to fasten together 40 sheets of paper—see Figure P2.46. Also, draw free-body diagrams for the handles and a diagram for the spring steel clip.

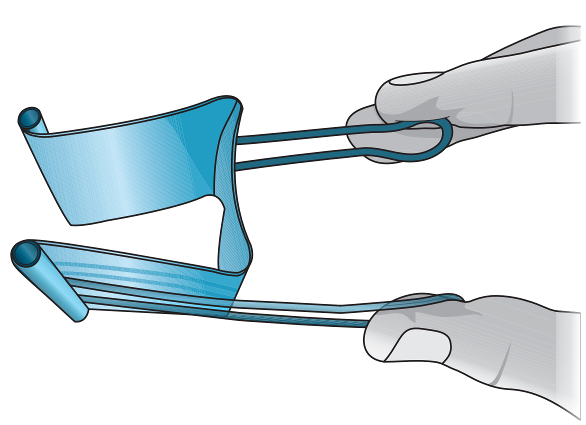


FIGURE P2.46

The handles are approximately 2.5 in. long and the handle wire is 1/16 in. in diameter. The spring clip (end view), when closed is shaped like a triangle. The triangle has two legs each 1 1/8 in. long and a third connecting side that is 1 in. long. The spring clip (side view) is approximately 2 in. wide.

2.47D The drawing in Figure P2.47D shows a electric squirrel cage blower supported by symmetric mountings at *A* and *B*. The motor delivers a torque of 1 N · m to the fan. Select a mounting width between 75 mm and 150 mm and then determine all loads acting on the blower. Sketch the free body in equilibrium.

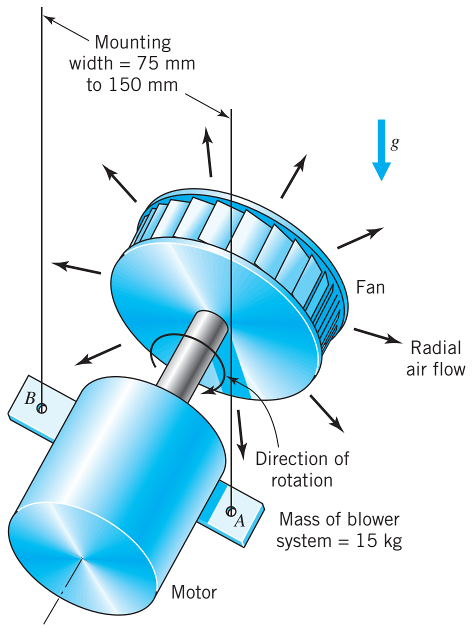


FIGURE P2.47D

2.48 Draw a free-body diagram for the gear and shaft assembly shown in Figure P2.48. Also sketch free-body diagrams for gear 1, gear 2, and the shaft.

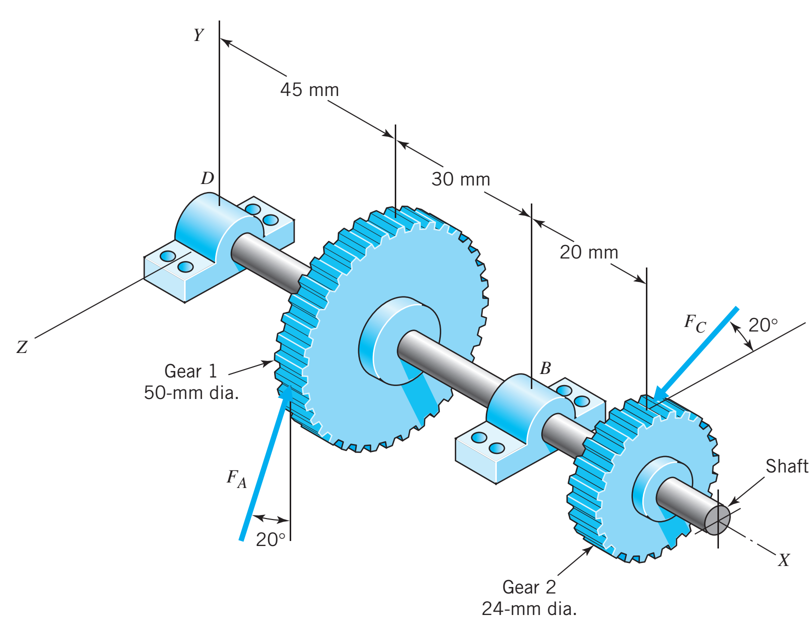


FIGURE P2.48

2.49 For the gear and shaft assembly shown in Figure P2.48, where $F_A = 550$ N, determine the magnitude of the force F_C. List assumptions.

2.50 For the gear and shaft assembly shown in Figure P2.48, where $F_A = 1000$ N, determine the forces at bearing *D*.

2.51 For the gear and shaft assembly shown in Figure P2.48, where $F_C = 750$ N, determine the forces at bearing *B*.

Section 2.3

2.52 The solid continuous member shown in Figure P2.52 can be viewed as composed of several straight segments. Draw free-body diagrams for the straight segments 1, 2, and 3 of Figure P2.52. Also, determine the magnitudes (symbolically) of the force and moments acting on the straight segments. Neglect the weight of the member.

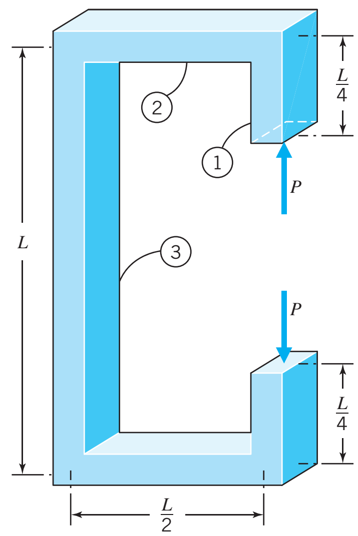


Figure P2.52

2.53 The drawings (Figure P2.53) show steel shafts supported by self-aligning bearings (which can provide radial but not bending loads to the shaft) at *A* and *B*. A gear (or a pulley or sprocket) causes each force to be applied as shown. Draw shear and bending moment diagrams neatly and to scale for each case. (Given dimensions are in millimeters.)

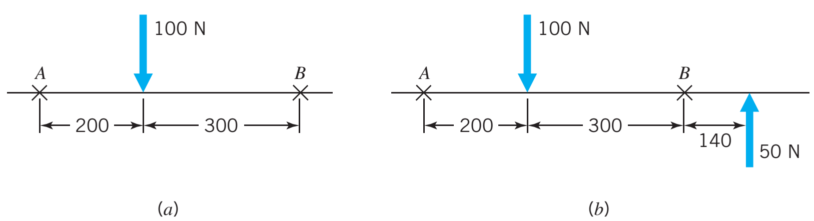


Figure P2.53

2.54 For each of the six cases shown in Figure P2.54, determine bearing reactions and draw appropriate shear and bending-moment diagrams for the 2-in.-dia. steel shaft supported by self-aligning ball bearings at *A* and *B*. A special 6-in.-pitch-diameter gear mounted on the shaft causes forces to be applied as shown.

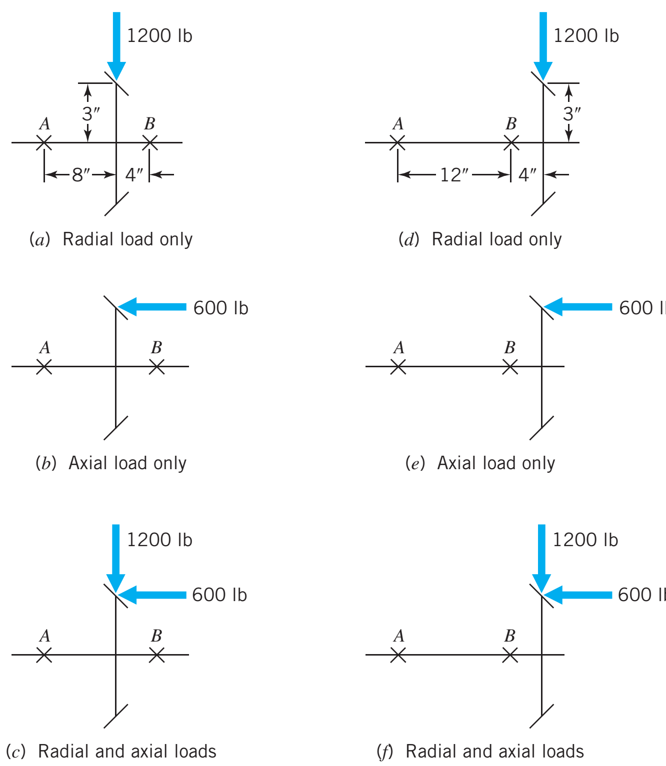


FIGURE P2.54

2.55 With reference to Figure P2.55

(a) Draw a free-body diagram of the structure supporting the pulley.

(b) Draw shear and bending moment diagrams for both the vertical and horizontal portions of the structure.

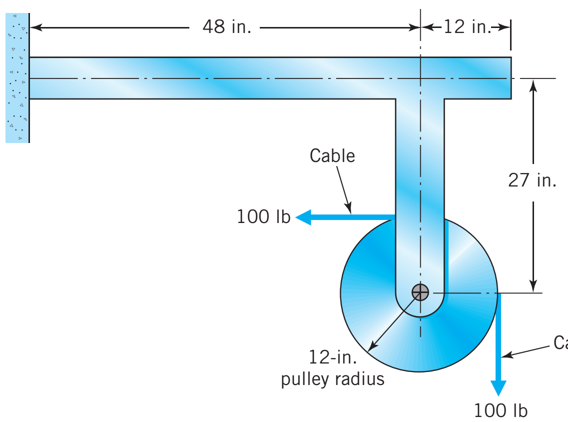


FIGURE P2.55

2.56 The drawing (Figure P2.56) shows a bevel gear attached to a shaft supported by self-aligning bearings at A and B and driven by a motor. Axial and radial components of the gear force are shown. The tangential or torque-producing component is perpendicular to the plane of the paper and has a magnitude of 2000 N. Bearing A takes thrust; B does not. Dimensions are in millimeters.

(a) Draw (to scale) axial load, shear, bending moment, and shaft torque diagrams.

(b) To what values of axial load and torque is the shaft subjected, and what portion(s) of the shaft experience these loads?

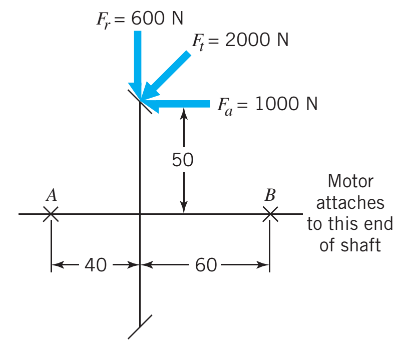


FIGURE P2.56

2.57 The shaft with bevel gear shown in Figure P2.57 is supported by self-aligning bearings A and B. (Given dimensions are in millimeters.) Only bearing A takes thrust. Gear loads in the plane of the paper are shown (the tangential or torque-producing force component is perpendicular to the paper). Draw axial load, shear, bending moment, and torque diagrams for the shaft.

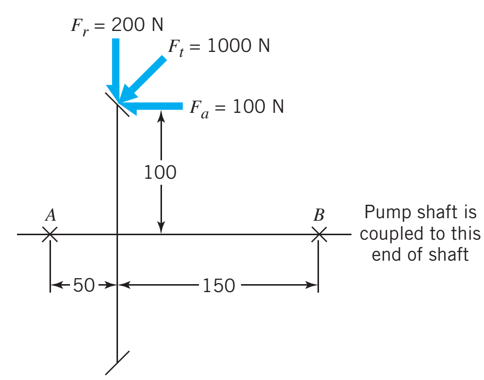


FIGURE P2.57

2.58 Same as Problem 2.57, except that the shaft in this drawing (Figure P2.58) has one bevel and one spur gear, and neither end of the shaft is connected to a motor or load.

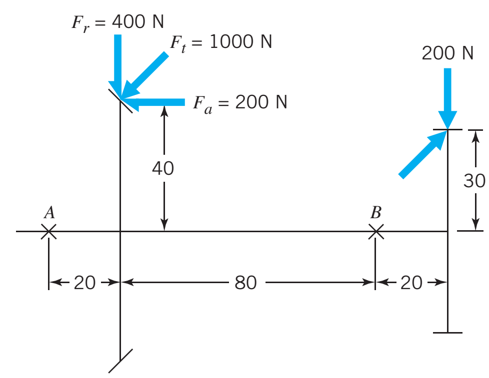


FIGURE P2.58

2.59 Same as Problem 2.57, except that the shaft in this drawing (Figure P2.59) has two bevel gears, and neither end of the shaft is connected to a motor or load.

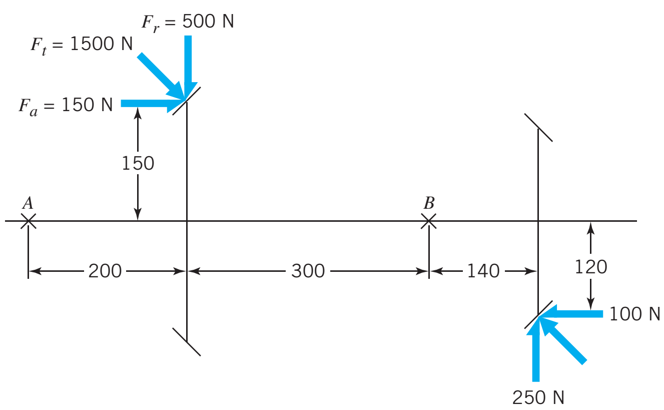


FIGURE P2.59

Section 2.4

2.60 Figure P2.60 shows a static force, F, applied to the tooth of a gear that is keyed to a shaft. Making appropriate simplifying assumptions, identify the stresses in the key, and write an equation for each.* State the assumptions made, and discuss briefly their effect.

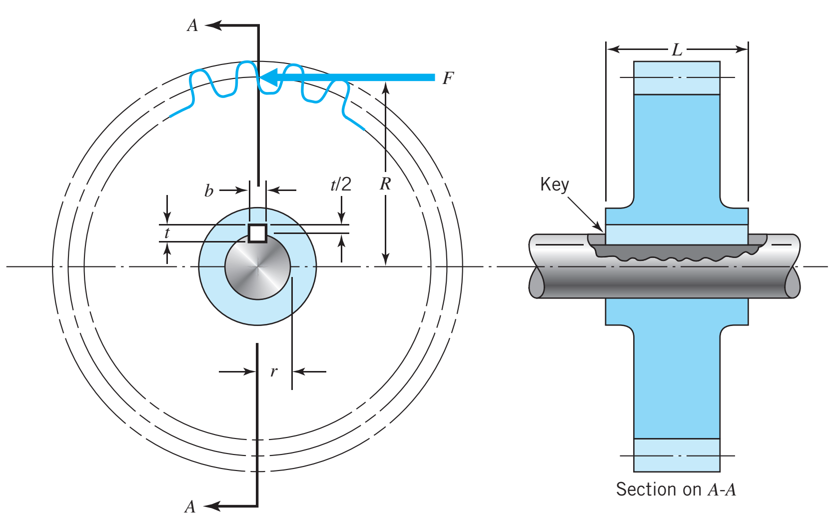


FIGURE P2.60

2.61 Figure P2.61 shows a screw with a square thread transmitting axial force F through a nut with n threads engaged (the drawing illustrates $n = 2$). Making appropriate simplifying assumptions, identify the stresses in the threaded portion of the screw and write an equation for each.* State the assumptions made, and discuss briefly their effect.

*The first five sections of Chapter 4 review simple stress equations.

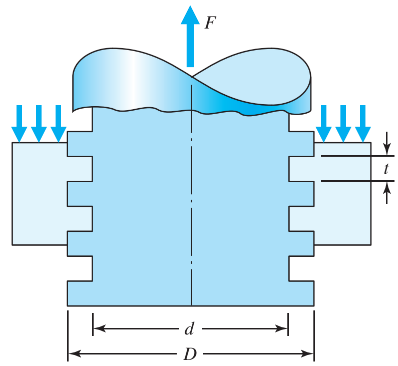


FIGURE P2.61

2.62 For the thin-walled, metal, cylindrical container, open on the right end and closed on the left end, where a force P is applied as shown in Figure P2.62, sketch the force flow lines. The container has a diameter of 3 in. and is 6 in. long. Using the force flow concept, locate the critical sections and/or surfaces for the container.

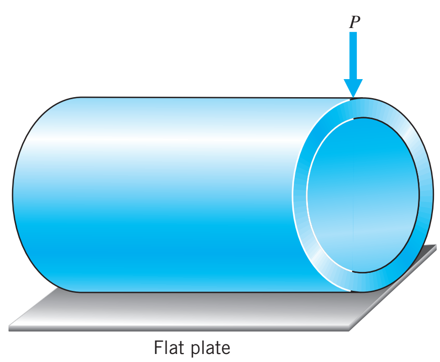


FIGURE P2.62

2.63 Figure P2.63 shows a total gas force F applied to the top of a piston.

(a) Copy the drawing and sketch the force paths through the piston, through the piston pin, and into the connecting rod.

(b) Making appropriate simplifying assumptions, identify the stresses in the piston pin and write an equation for each.* State the assumptions made, and discuss briefly their effect.

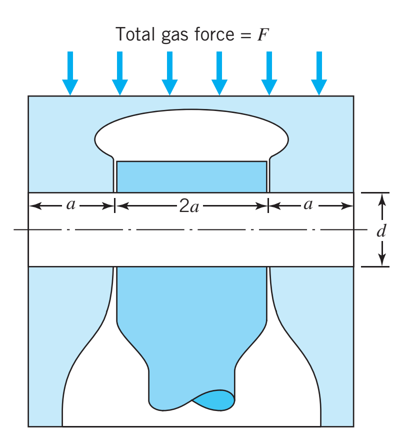


FIGURE P2.63

*The first five sections of Chapter 4 review simple stress equations.

2.64 Figure P2.64 shows force P applied to an engine crankshaft by a connecting rod. The shaft is supported by main bearings A and B. Torque is transmitted to an external load through flange F.

(a) Draw the shaft, and show all loads necessary to place it in equilibrium as a free body.

(b) Starting with P and following the force paths through the shaft to the flange, identify the locations of potentially critical stresses.

(c) Making appropriate simplifying assumptions, write an equation for each.* State the assumptions made.

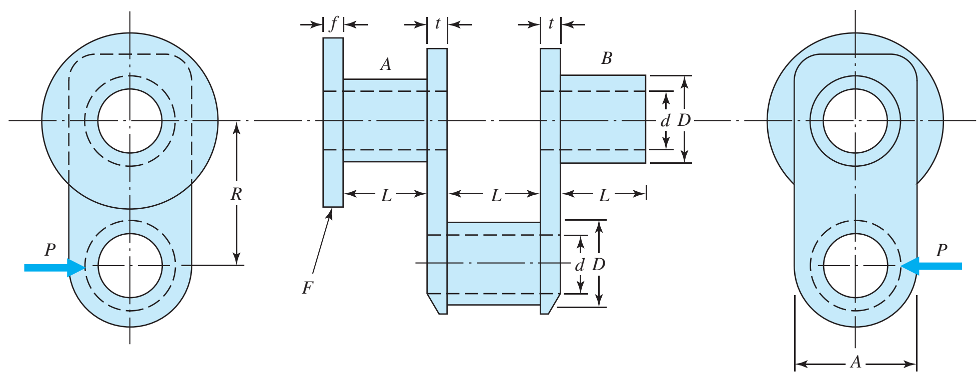


FIGURE P2.64

Section 2.5

2.65 In Figure P2.65, all the joints are pinned and all links have the same length L and the same cross-sectional area A. The central joint (pin) is loaded with a force P. Determine the forces in the bars.

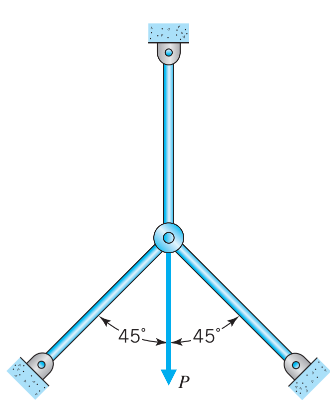


FIGURE P2.65

2.66 Repeat Problem 2.65, except where the top link has a cross-sectional area of A, and the two lower links have a cross-sectional area of A'. Determine (a) the force in the bars, and (b) the ratio A/A' that will make the force in all the links numerically equal.

*The first five sections of Chapter 4 review simple stress equations.

2.67 A “T” bracket, attached to a fixed surface by four bolts, is loaded at point *E* as shown in Figure P2.67.

(a) Copy the drawing and sketch paths of force flow going to each bolt.

(b) If the stiffness between point *E* and the plate through bolts *B* and *C* is twice the stiffness between point *E* and the plate through bolts *A* and *D*, how is the load divided between the four bolts?

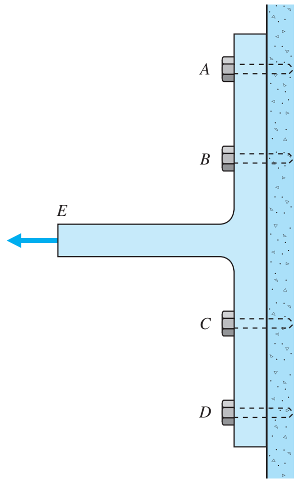


FIGURE P2.67

2.68 A very stiff horizontal bar, supported by four identical springs, as shown in Figure P2.68, is subjected to a center load of 100 N. What load is applied to each spring? [Ans.: lower springs, 40 N; upper springs, 20 N]

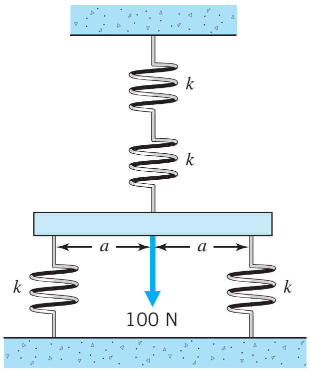


FIGURE P2.68

2.69 Repeat Problem 2.68, except assume that the horizontal bar as configured is not rigid and also has a spring constant of *k*.

Section 2.6

2.70 With reference to the bolts in Problem 2.67,

(a) If they are brittle and each one fractures at a load of 6000 N, what maximum force *F* can be applied to the bracket?

(b) What load can be applied if they are ductile, and each bolt has a yield strength of 6000 N?

2.71 Figure P2.71 shows two plates joined with straps and a single row of rivets (or bolts). Plates, straps, and rivets are all made of ductile steel having yield strengths in tension, compression, and shear of 284, 284, and 160 MPa, respectively. Neglect frictional forces between the plates and straps.

(a) What force *F* can be transmitted across the joint per pitch, *P*, of joint width, based on rivet shear strength?

(b) Determine minimum values of *t*, *t*′, and *P* that will permit the total joint to transmit this same force (thus giving a "balanced" design).

(c) Using these values, what is the "efficiency" of the joint (ratio of joint strength to strength of a continuous plate)?

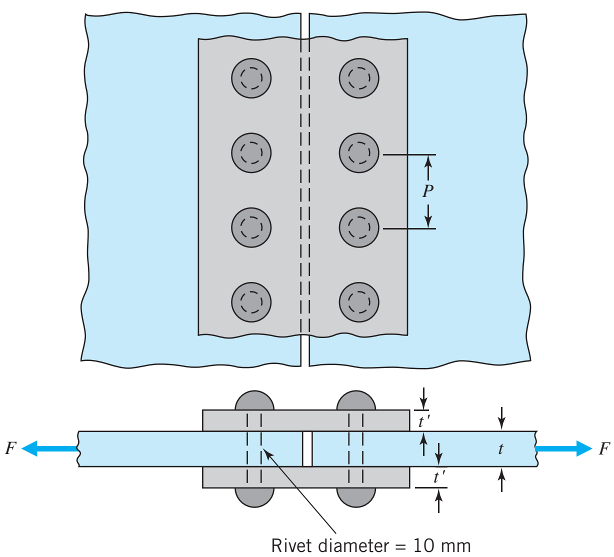


FIGURE P2.71

2.72 Plates 20 mm thick are butted together and spliced using straps 10 mm thick and rivets (or bolts) of 40-mm diameter. A double-riveted joint is used, and this is exactly as shown in Figure 2.19 except that the inner row of rivets is eliminated on both sides. All materials have tensile, compression, and shear yield strengths of 200, 200, and 120 MPa, respectively. Neglect friction between plates and straps. Determine the pitch, *P*, giving the greatest joint strength. How does this compare with the strength of a continuous plate?